
More exercises on maximal solutions

1 Exercise 1

We consider the differential equation

$$y'(t) = y(t) \sin^2(y(t))$$

1. What are the constant functions that are solutions of this differential equation?
2. Let y be a maximal solution satisfying $y(0) = y_0$. Prove that y is bounded and monotonous.
3. Prove that y is defined over \mathbb{R} .

2 Exercise 2

We consider the differential equation

$$y'(t) = \cos(y(t)) + \frac{1}{2} \sin(t)$$

1. With a graphical analysis, find some horizontal barriers. Prove that they are indeed barriers.
2. Let f be the solution satisfying the initial condition $f(0) = 0$. Prove that f is bounded and defined over \mathbb{R} .
3. Prove that every solution is bounded and defined over \mathbb{R} .