Epidemiology : modeling of a cold epidemy

This exercise aims at studying the SIS epidemiological model. This model is notably applied for mild diseases as a cold, for which infected individuals very rarely die but can be infected several times. Let us denote N the total population, assumed constant along the time of the study, and $y(t)$ the number of individuals infected at time t. We assume that the variation dy of y during an infinitesimal time dt is caused by the following events :

- the new contaminations, assumed proportional to the product $y(N-y)$, with a constant positive coefficient α,
- the healings , assumed proportional to y, with a constant positive coefficient β.

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1. Explain why such a modelling has been chosen, in particular the term $y(N-y)$.

2. Prove that the differential equation corresponding to this model is

$$y'(t) = (\alpha N - \beta)y(t) - \alpha y^2(t).$$

3. Solve this equation. You can use the change of variables $u = \dfrac{1}{y}$. What is the influence of the sign of $\alpha N - \beta$? How does look the graph of the solution in the case $\alpha N - \beta > 0$?

4. The table hereafter indicates the total number of colds in a small town during December and January, as a function of the number of past days. For instance, on 28th day, there were 1940 colds. Assuming that the SIS model is relevant, and $\alpha = 2.382.10^{-5}$ and $\beta = 0.1572$, use this table to provide an approximate previsional number of the maximum of the total number of colds during one day.

20	22	24	26	28	30	32	34	36	38	40	42	44
480	697	1000	1408	1940	2600	3366	4194	5014	5791	6452	6986	7396

5. Give an estimation of the number of residents in the town.