
Geometrical study of ODE's exercises

1 Exercise 1

We consider the differential equation $y' = -xy$. We aim to get the general look of the tangent field for this equation.

1. What is the formula giving the slope $\phi(x, y)$ of the tangent field on a point (x, y) ?
2. Draw the tangent field at points $(-2, 0)$, $(-1, 0)$, $(0, 0)$, $(1, 0)$ and $(2, 0)$. Find a simple function solution of the differential equation.
3. Draw the tangent field on some points of the y -axis.
4. What is the sign of the slope of the tangent field on a point (x, y) with $x > 0$ and $y > 0$? Draw the field on points $(1, 1)$, $(1, 2)$, $(2, 1)$ and $(2, 2)$.
5. Similarly, draw the tangent field on some points (x, y) with $x < 0$ and $y > 0$.
6. Draw approximately the shape of the solution for the initial condition $y(0) = 2$.

2 Exercise 2

We consider the differential equation $y' = 2x - y$. We aim to get the general look of the tangent field for this equation.

1. On which set of points of the plane is the tangent horizontal? Similarly, find the set of points where the slope of the tangent is negative, then positive.
2. What is the slope on point $(0, 2)$? Draw approximately the graph of the solution for the initial condition $y(0) = 2$.
3. Find the set of points where the slope is 2. Deduce another solution. Check with the calculus if this solution is right.
4. Draw the shape of the tangent field.
5. Check that for every constant C , the functions $f(x) = C \exp(-x) + 2(x - 1)$ are solutions of the equation.