Geometrical study of ODE's

(These notes are strongly inspired by the course of Frédéric Le Roux, at University Paris 11)

We consider a first order ODE written under the following form :

$$y' = f(x, y).$$

We aim to study geometrically the solutions of this ODE.

We define on every point (x, y) of the plane (at least, where the function f is defined), the vector $V_{x,y}$ whose slope is $f(x, y)$.

Proposition 0.1. *We denote u a solution of the differential equation $y' = f(x, y)$. Then the graph of u is tangent on each of its points (x, y) to the vector $V_{x,y}$.*

With this property one can get an idea of the behavior of the solutions without explicitly solving the differential equation.

For instance, on Fig. 1 are plotted some vectors $V_{x,y}$ for $f(x, y) = -0.5y$. On Fig 2 are plotted some vectors $V_{x,y}$ for $f(x, y) = (2 - cos(x))y - 0.5y^2 - 1$, with the graph of a solution.

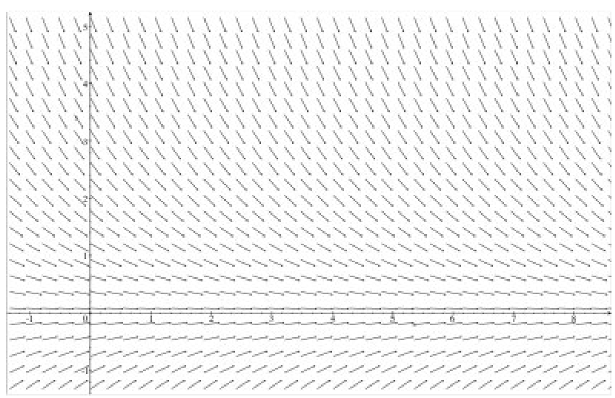

FIGURE 1 – Example of a tangent field, for $f(x, y) = -0.5y$

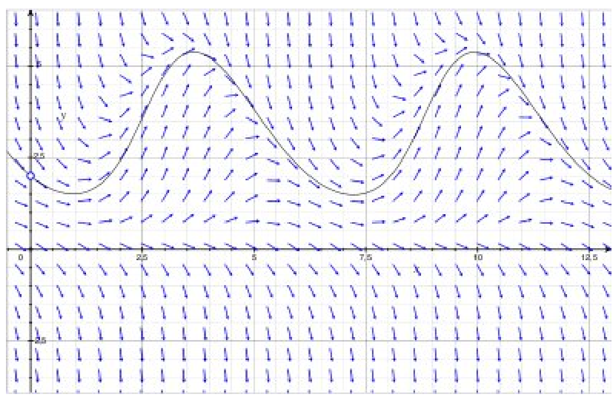

FIGURE 2 – Example of a tangent field for $f(x, y) = (2 - cos(x))y - 0.5y^2 - 1$, with the graph of a solution.