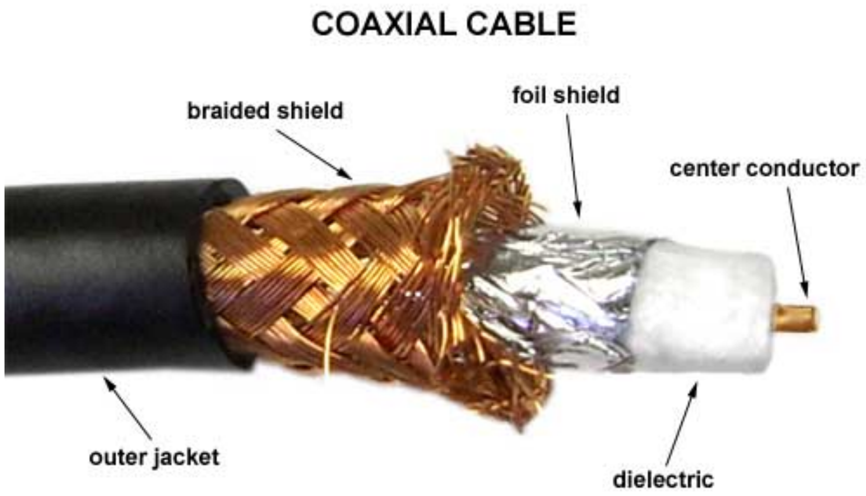
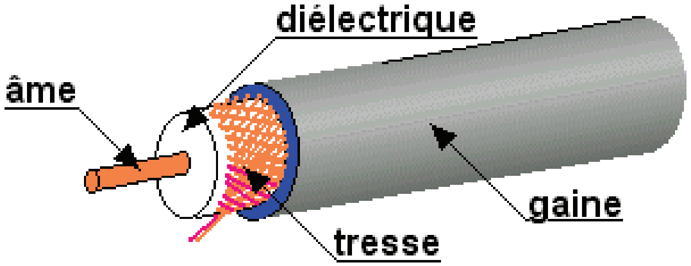
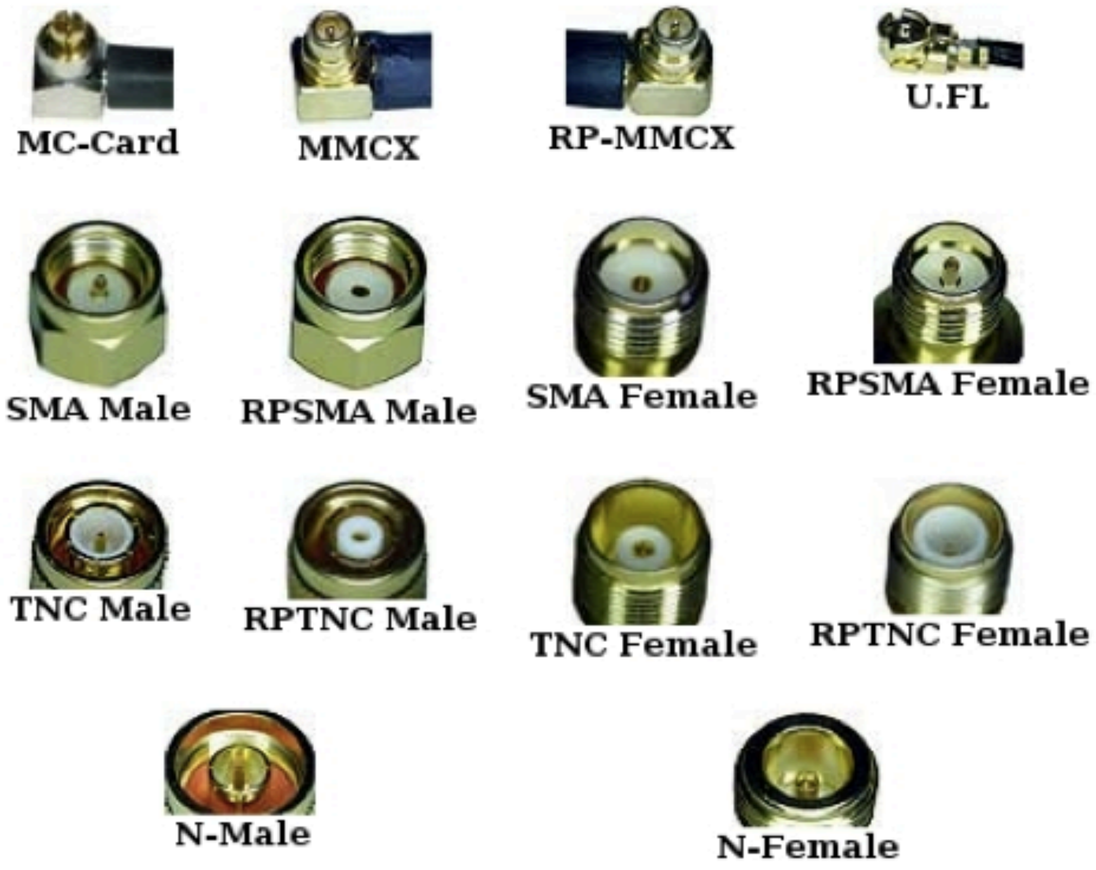
Propagation in coaxial transmission lines





Connectors

Les connecteurs présentent une grande variété de formes et de tailles. En plus de types standard, les connecteurs peuvent être de ***polarité inverse*** (sexes inversés) ou de ***filetée inverse***.



Adaptators and Pigtails

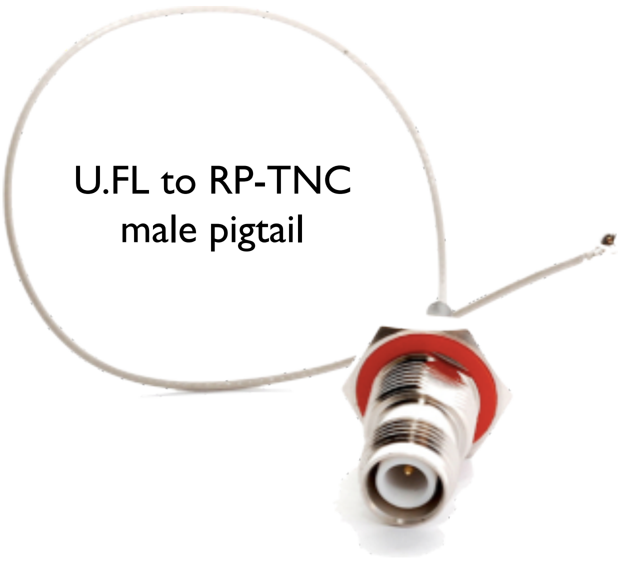
Les adaptateurs et pigtails sont utilisés pour interconnecter les différents types de câbles ou de dispositifs.


SMA female to N male


N male to N male


N female to N female


SMA male to TNC male


U.FL to RP-TNC male pigtail


U.FL to N male pigtail


SMA male to N female

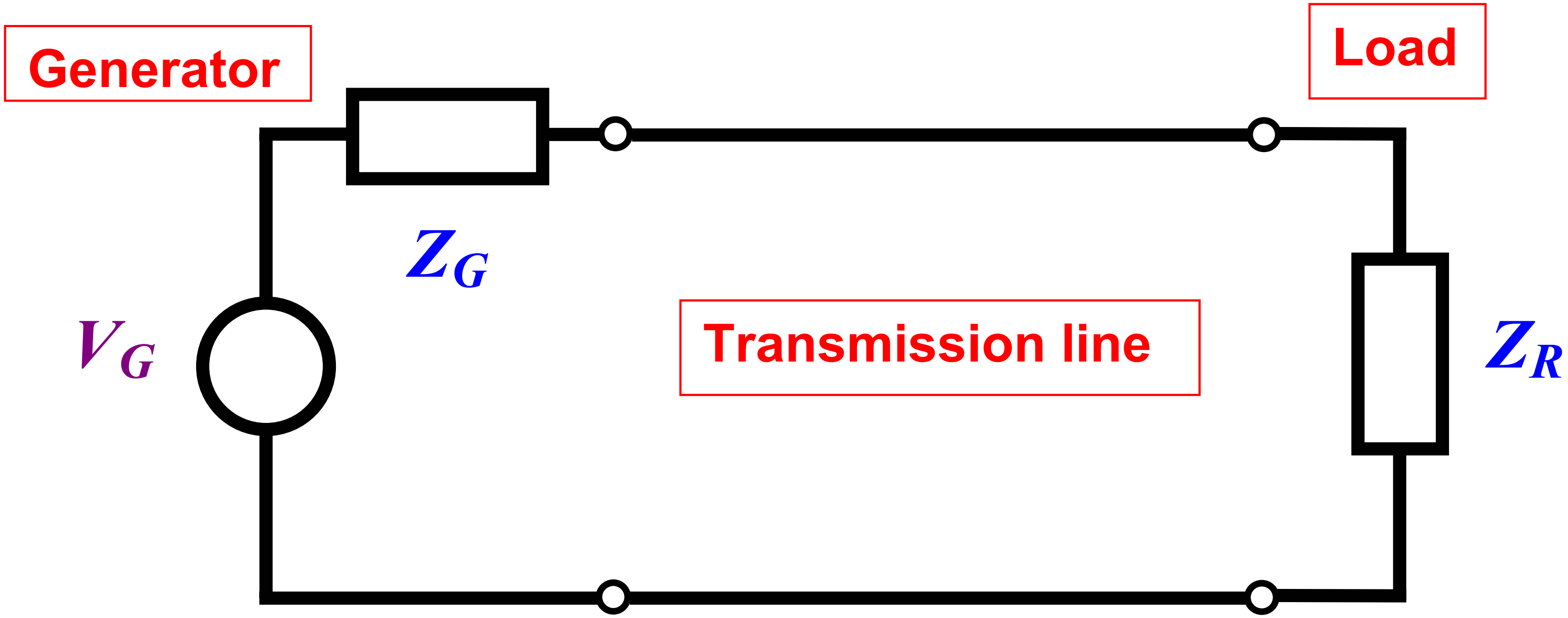
Losses

La perte (ou ***atténuation***) d'un câble coaxial dépend de la construction du câble et la fréquence de fonctionnement. Le montant total de la perte est proportionnelle à la longueur du câble.

type de cable	diametre	atténuation à 2.4 GHz	atténuation à 5.3 GHz
RG-58	4.95 mm	0.846 dB/m	1.472 dB/m
RG-213	10.29 mm	0.475 dB/m	0.829 dB/m
LMR-400	10.29 mm	0.217 dB/m	0.341 dB/m
LDF4-50A	16 mm	0.118 dB/m	0.187 dB/m

http://www.ocarc.ca/coax.htm

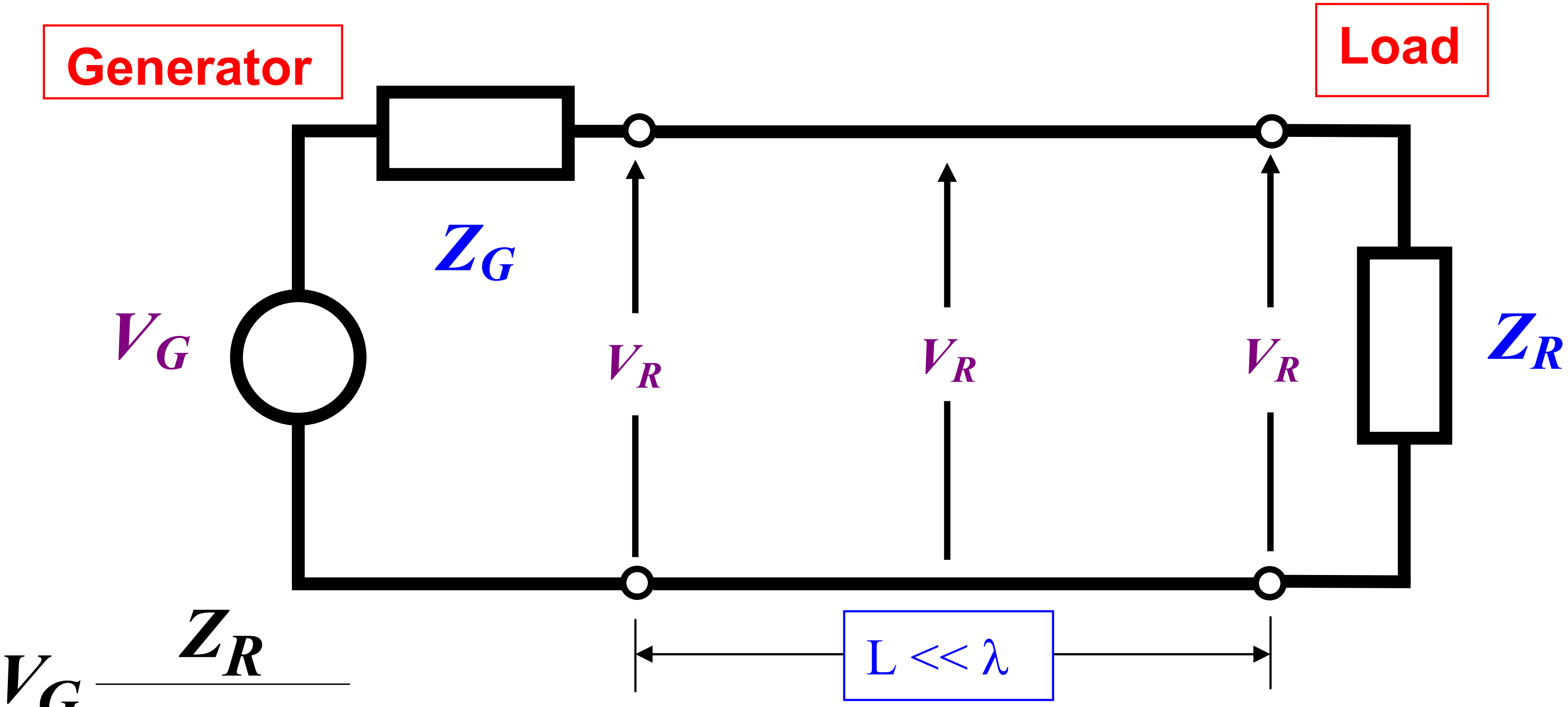
1. Problem statement



Outline

1. Problem Statement
2. Looking for a equivalent network
3. Propagation in a line
4. Maximal power transmission
5. Transient propagation (case studies)

Case 1: L<<λ



$$V_R = V_G \frac{Z_R}{Z_G + Z_R}$$

Orders of magnitudes

Let's look at some examples. The electricity supplied to households consists of high power sinusoidal signals, with frequency of 60Hz or 50Hz, depending on the country. Assuming that the insulator between wires is air ($\varepsilon \approx \varepsilon_0$), the wavelength for 60Hz is:

$$\lambda = \frac{c}{f} = \frac{2.999 \times 10^8}{60} \approx 5.0 \times 10^6 \, m = 5,000 \, km$$

which is the about the distance between S. Francisco and Boston!

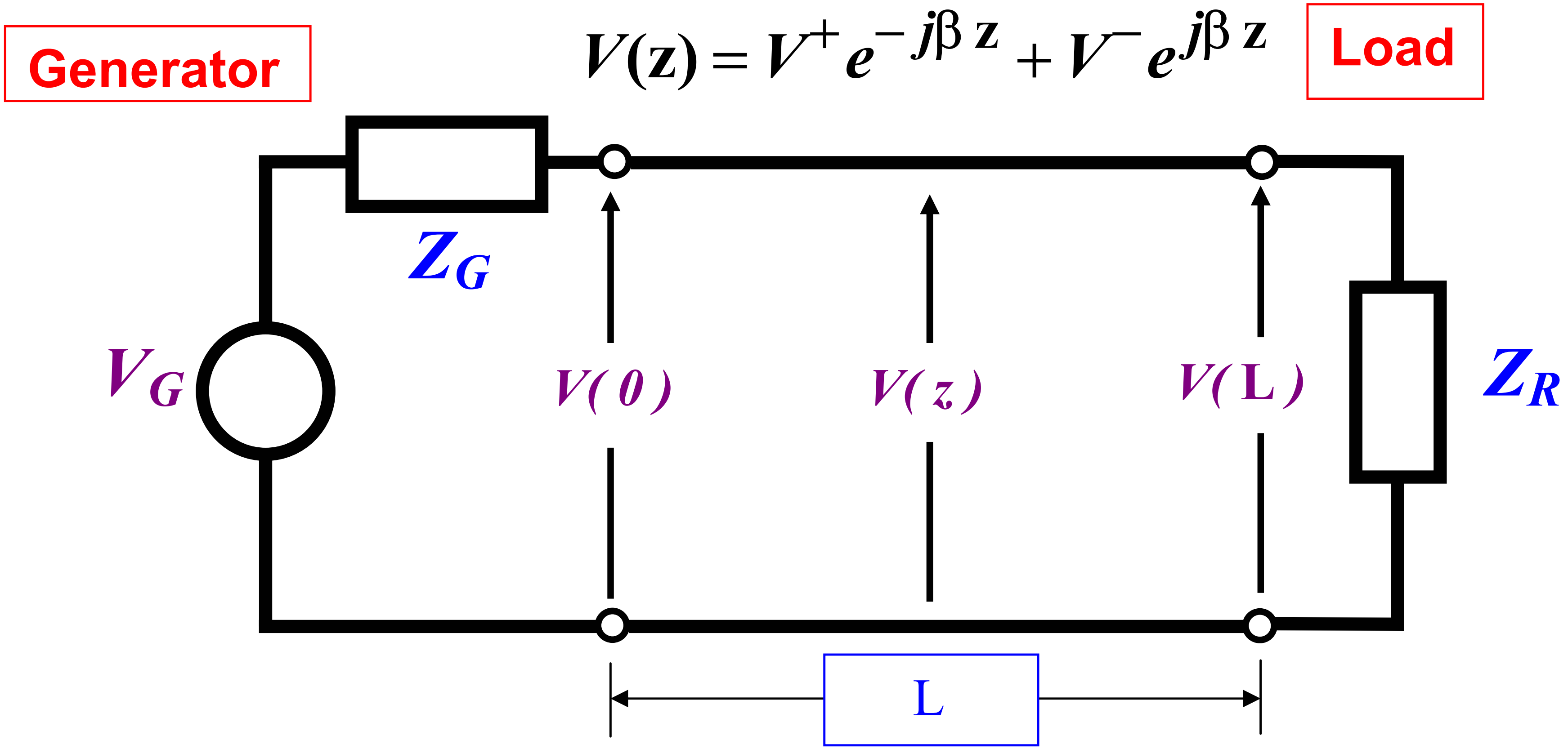
Let's compare to a frequency in the microwave range, for instance 60 GHz. The wavelength is given by

$$\lambda = \frac{c}{f} = \frac{2.999 \times 10^8}{60 \times 10^9} \approx 5.0 \times 10^{-3} \, m = 5.0 \, mm$$

which is comparable to the size of a microprocessor chip.

Which conclusions do you draw?

Case 2 : L>>λ



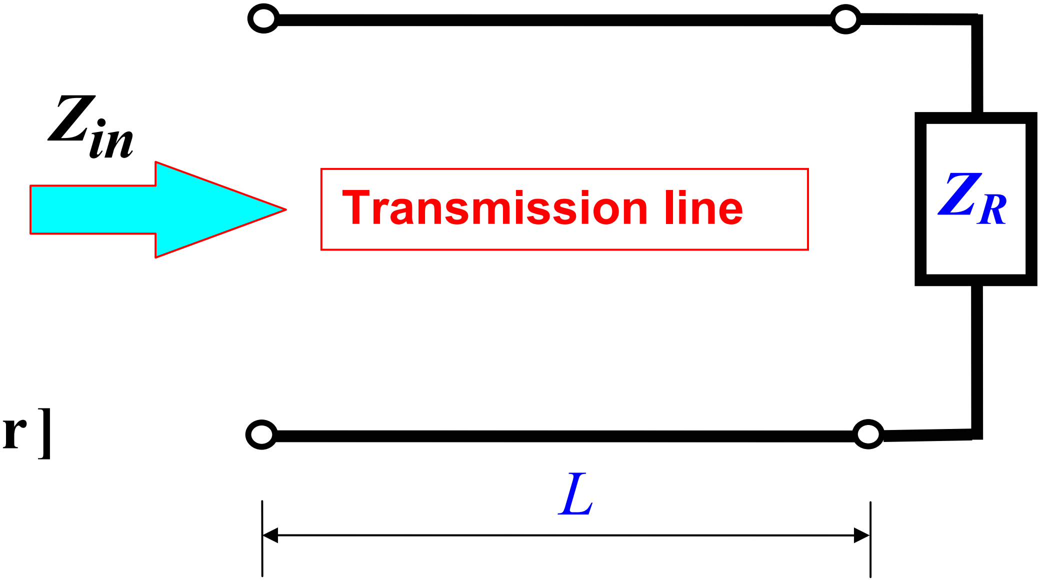
$$V(z) = V^{+} e^{-j\beta z} + V^{-} e^{j\beta z}$$

The simplest circuit problem that we can study consists of a voltage generator connected to a load through a uniform transmission line. In general, the impedance seen by the generator is not the same as the impedance of the load, because of the presence of the transmission line, except for some very particular cases:

$$Z_{in} = Z_R$$

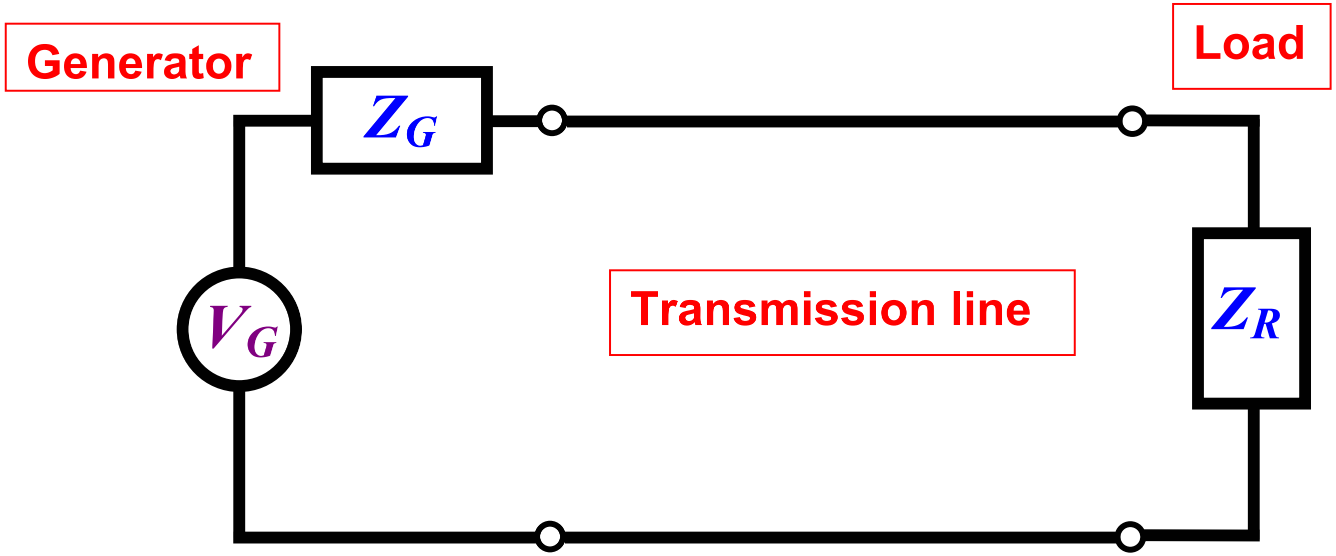
only if

$$L = n\frac{\lambda}{2} \qquad [\,n = \text{integer}\,]$$



Our first goal is to determine the equivalent impedance seen by the generator, that is, the input impedance of a line terminated by the load. Once that is known, standard circuit theory can be used.

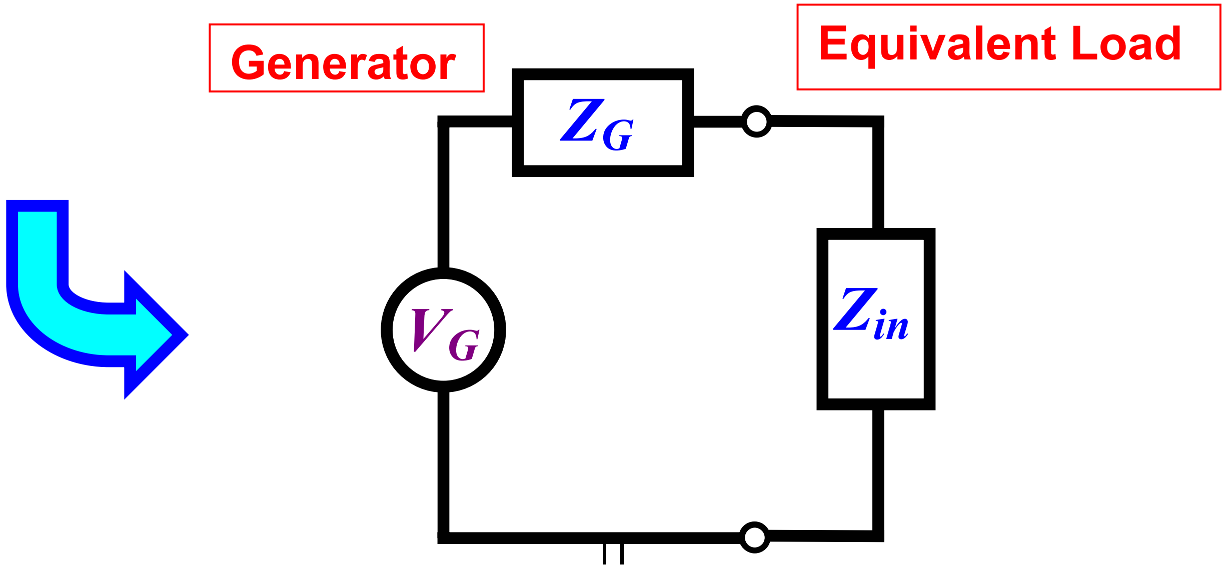
2. Looking for an equivalent network



Generator

Load

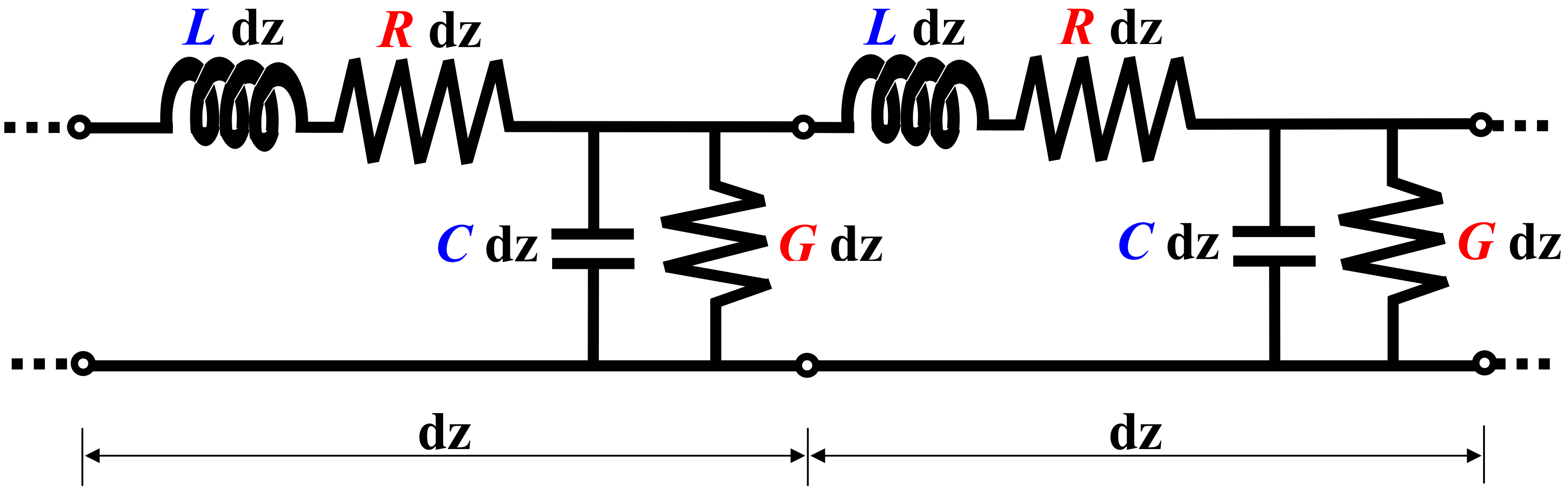
Transmission line



Generator

Equivalent Load

Line equivalent network

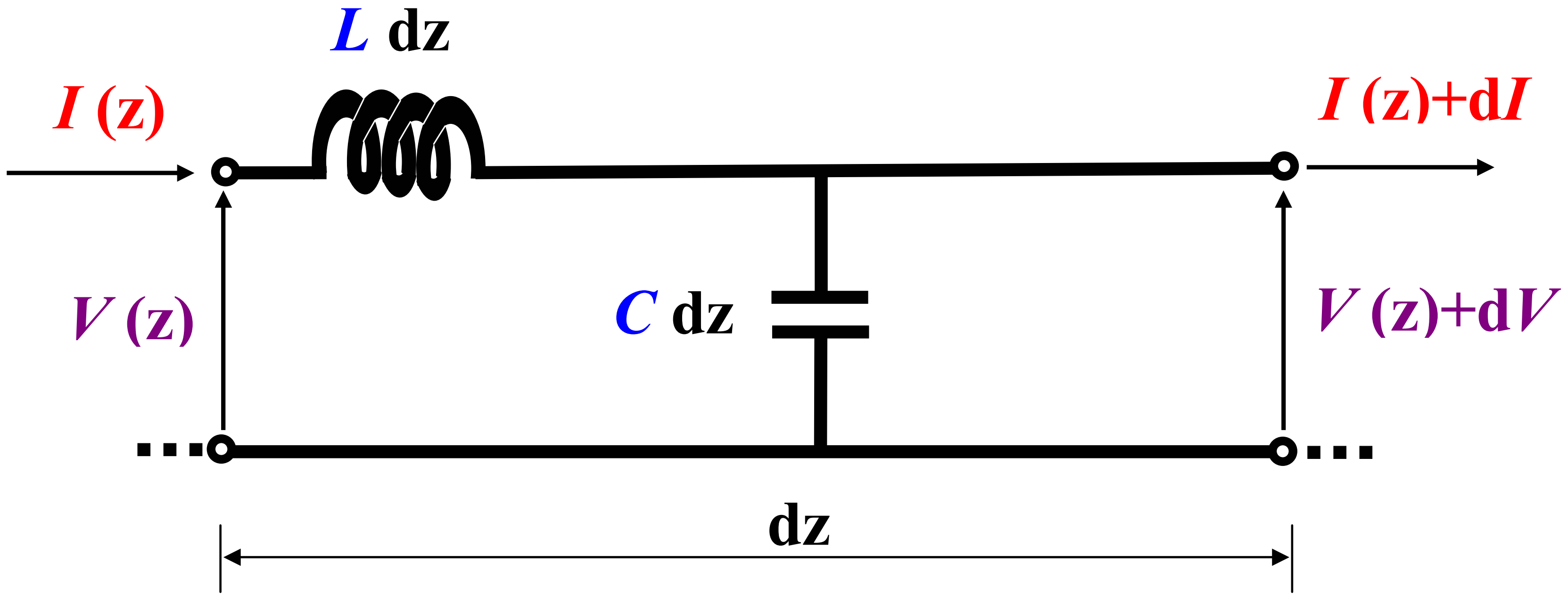


The impedance parameters *L*, *R*, *C*, and *G* represent:

L **= series inductance per unit length**
R **= series resistance per unit length**
C **= shunt capacitance per unit length**
G **= shunt conductance per unit length.**

Each cell of the distributed circuit will have impedance elements with values: *L*dz, *R*dz, *C*dz, and *G*dz, where dz is the infinitesimal length of the cells.

Without losses



Signal propagation is quantified in terms of the solution of the so-called Telegrapher's equations

$$\frac{dI}{dz} = -j\omega C V$$

$$\frac{d^2V}{dz^2} = -j\omega L \frac{dI}{dz} = j\omega L \; j\omega CV = -\omega^2 LC V$$

$$\frac{d^2I}{dz^2} = -j\omega C \frac{dV}{dz} = j\omega C \; j\omega L I = -\omega^2 LC I$$

$$\frac{dV}{dz} = -j\omega L I$$

The general solution for the voltage equation is

$$V(z) = V^{+} e^{-j\beta z} + V^{-} e^{j\beta z}$$

where the wave propagation constant is

$$\beta = \omega\sqrt{LC}$$

We have the following useful relations:

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \frac{\omega}{v_p}$$

$$= \frac{\omega\sqrt{\varepsilon_r \mu_r}}{c} = \omega\sqrt{\varepsilon_0 \mu_0}\sqrt{\varepsilon_r \mu_r} = \omega\sqrt{\varepsilon \mu}$$

Here, $\lambda = v_p / f$ is the wavelength of the dielectric medium surrounding the conductors of the transmission line and

$$v_p = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} = \frac{1}{\sqrt{\varepsilon \mu}}$$

is the phase velocity of an electromagnetic wave in the dielectric.

As you can see, the propagation constant β can be written in many different, equivalent ways.

The current distribution on the transmission line can be readily obtained by differentiation of the result for the voltage

$$\frac{\mathrm{d}V}{\mathrm{dz}} = -j\beta V^{+} e^{-j\beta z} + j\beta V^{-} e^{j\beta z} = -j\omega L\ I$$

which gives

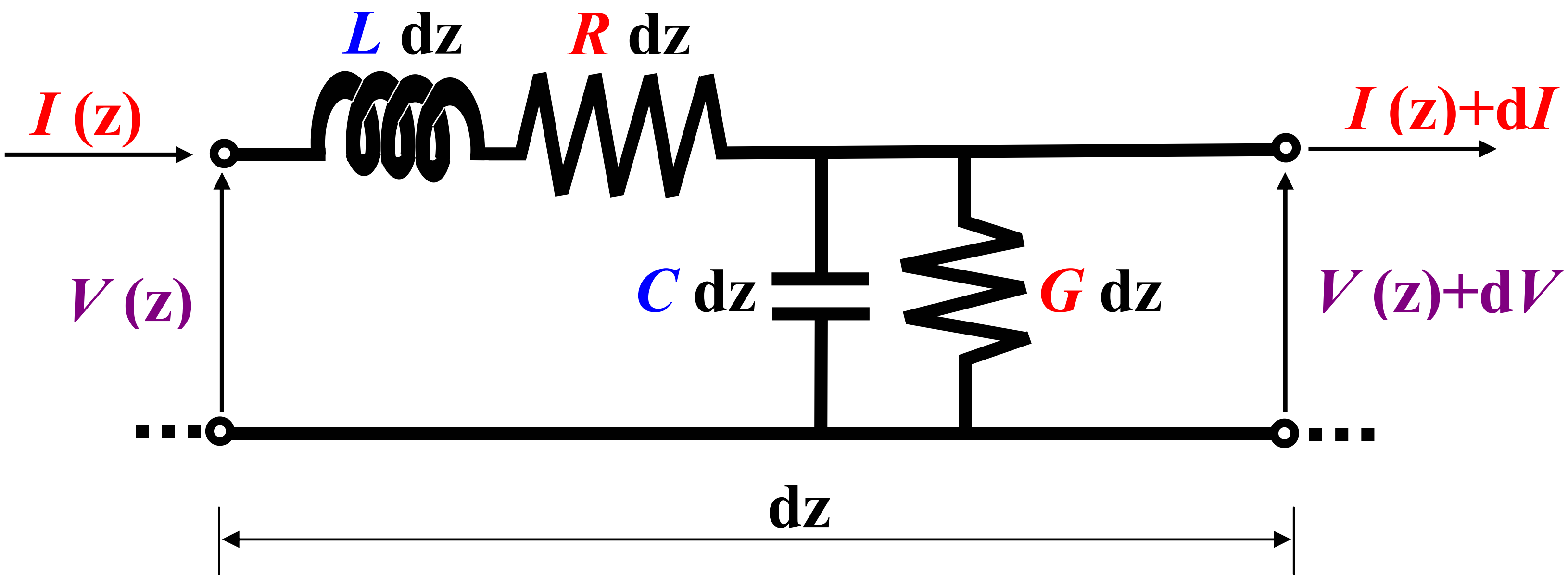
$$I(z) = \sqrt{\frac{C}{L}}\left(V^{+} e^{-j\beta z} - V^{-} e^{j\beta z}\right) = \frac{1}{Z_0}\left(V^{+} e^{-j\beta z} - V^{-} e^{j\beta z}\right)$$

The real quantity

$$Z_0 = \sqrt{\frac{L}{C}}$$

is the “characteristic impedance” of the loss-less transmission line.

With losses



Telegrapher's equations with losses

$$\frac{\mathrm{d}I}{\mathrm{d}z} = -(j\omega C + G)V$$

$$\frac{\mathrm{d}^2 V}{\mathrm{d}z^2} = -(j\omega L + R)\frac{\mathrm{d}I}{\mathrm{d}z} = (j\omega L + R)(j\omega C + G)V$$

$$\frac{\mathrm{d}^2 I}{\mathrm{d}z^2} = -(j\omega C + G)\frac{\mathrm{d}V}{\mathrm{d}z} = (j\omega C + G)(j\omega L + R)I$$

$$\frac{\mathrm{d}V}{\mathrm{d}z} = -(j\omega L + R)I$$

with the "characteristic impedance" of the lossy transmission line

$$Z_0 = \sqrt{\frac{(j\omega L + R)}{(j\omega C + G)}}$$

Note: the characteristic impedance is now complex !

Common mistakes

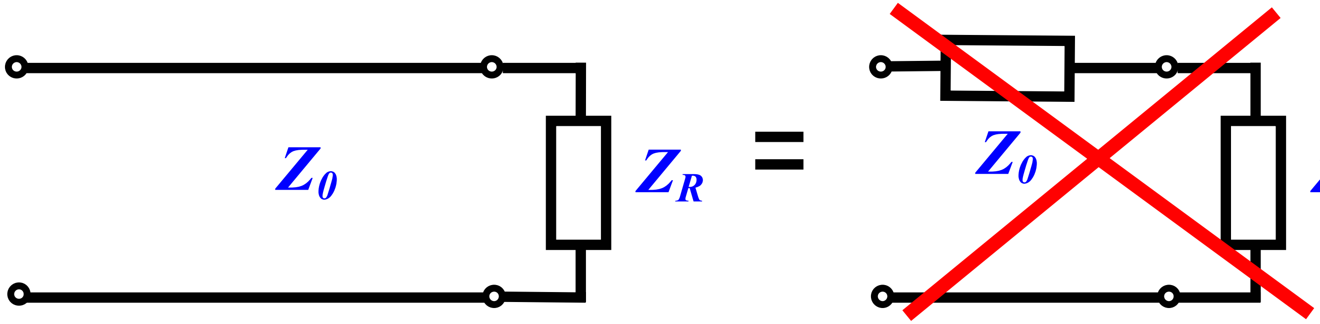
For both loss-less and lossy transmission lines

the characteristic impedance does not depend on the line length

but only on the metal of the conductors, the dielectric material surrounding the conductors and the geometry of the line cross-section, which determine L, R, C, and G.

One must be careful not to interpret the characteristic impedance as some lumped impedance that can replace the transmission line in an equivalent circuit.

This is a very common mistake!



3. Propagation in a line

We have obtained the following solutions for the steady-state voltage and current phasors in a transmission line:

Loss-less line

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z}$$

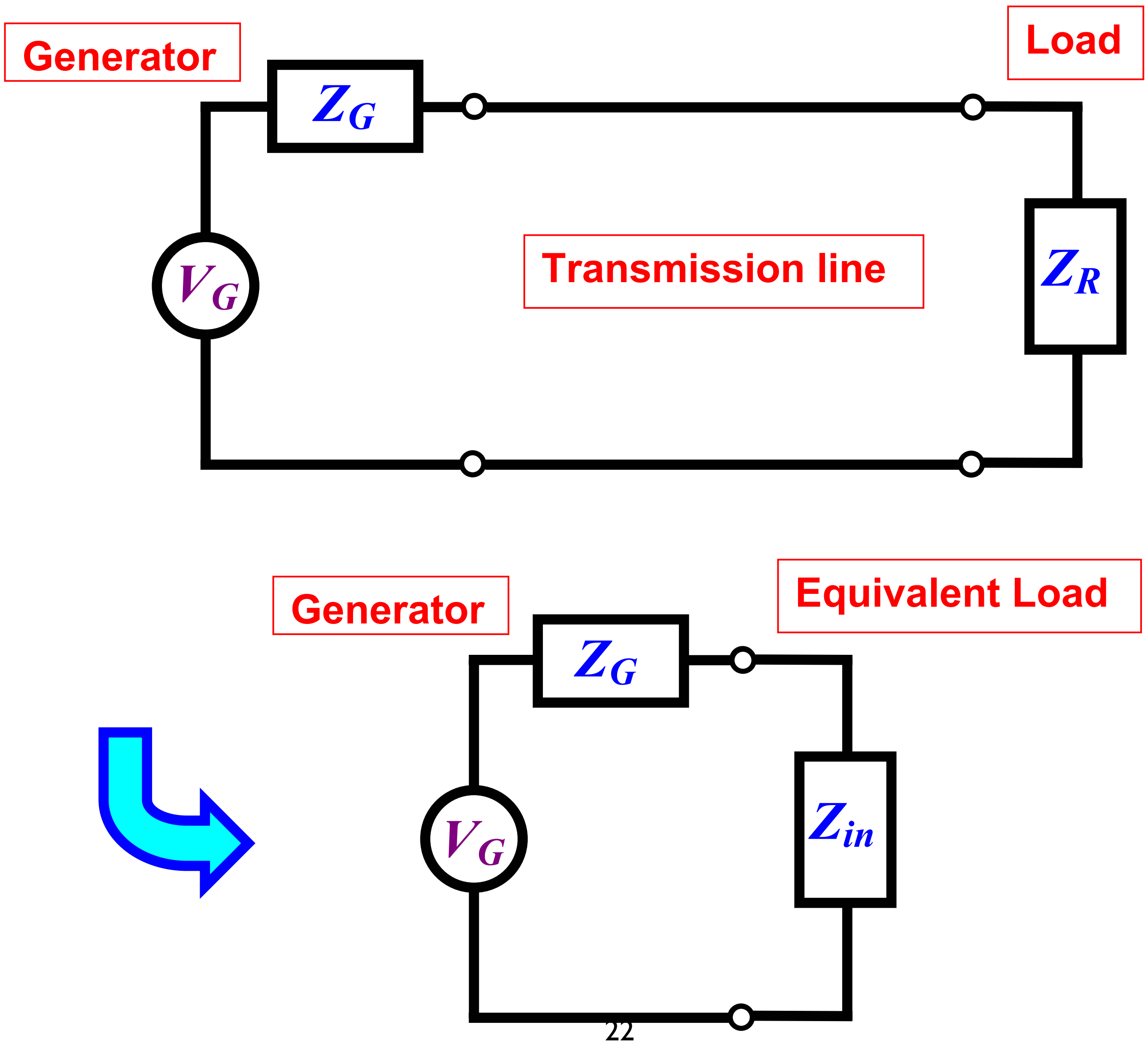
$$I(z) = \frac{1}{Z_0}\left(V^{+}e^{-j\beta z} - V^{-}e^{j\beta z}\right)$$

Lossy line

$$V(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z}$$

$$I(z) = \frac{1}{Z_0}\left(V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right)$$

General case



Generator

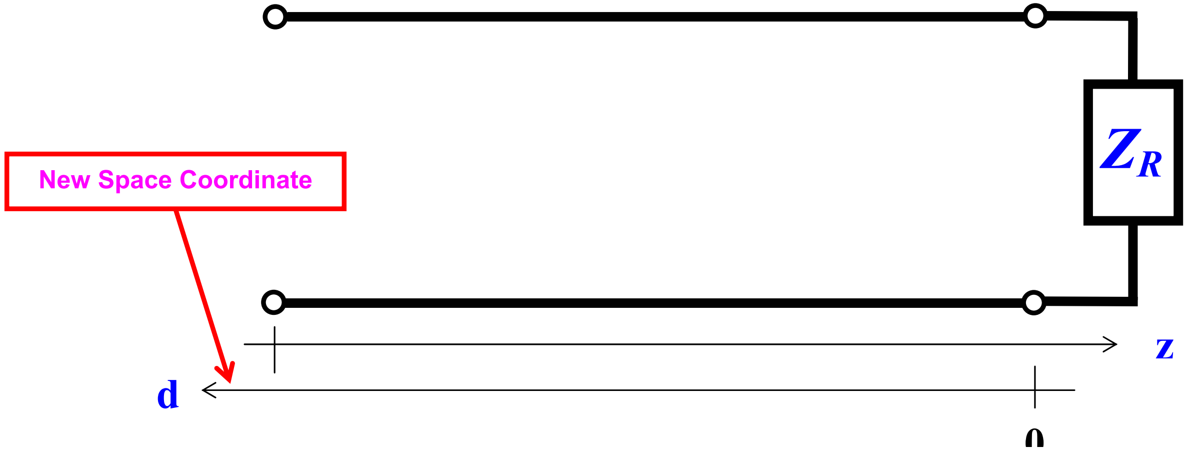
Load

Transmission line

Generator

Equivalent Load

Before we consider the boundary conditions, it is very convenient to shift the reference of the space coordinate so that the zero reference is at the location of the load instead of the generator. Since the analysis of the transmission line normally starts from the load itself, this will simplify considerably the problem later.



New Space Coordinate

We adopt a new coordinate $\mathbf{d} = -\,\mathbf{z}$, with zero reference at the load location. The new equations for voltage and current along the lossy transmission line are

Loss-less line

$$V(\mathrm{d}) = V^{+}e^{j\beta \mathrm{d}} + V^{-}e^{-j\beta \mathrm{d}}$$

$$I(\mathrm{d}) = \frac{1}{Z_0}\left(V^{+}e^{j\beta \mathrm{d}} - V^{-}e^{-j\beta \mathrm{d}}\right)$$

Lossy line

$$V(\mathrm{d}) = V^{+}e^{\gamma \mathrm{d}} + V^{-}e^{-\gamma \mathrm{d}}$$

$$I(\mathrm{d}) = \frac{1}{Z_0}\left(V^{+}e^{\gamma \mathrm{d}} - V^{-}e^{-\gamma \mathrm{d}}\right)$$

At the load ($\mathbf{d=0}$) we have, for both cases,

$$V(0) = V^+ + V^-$$

$$I(0) = \frac{1}{Z_0}\left(V^+ - V^-\right)$$

For a given load impedance Z_R, the load boundary condition is

$$V(0) = Z_R\, I(0)$$

Therefore, we have

$$V^+ + V^- = \frac{Z_R}{Z_0}\left(V^+ - V^-\right)$$

from which we obtain the voltage load reflection coefficient

$$\boxed{\Gamma_R = \frac{V^-}{V^+} = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

We can introduce this result into the transmission line equations as

Loss-less line

$$V(\mathrm{d}) = V^{+} e^{j\beta \mathrm{d}} \left(1 + \Gamma_R\, e^{-2j\beta \mathrm{d}}\right)$$

$$I(\mathrm{d}) = \frac{V^{+} e^{j\beta \mathrm{d}}}{Z_0} \left(1 - \Gamma_R e^{-2j\beta \mathrm{d}}\right)$$

Lossy line

$$V(\mathrm{d}) = V^{+} e^{\gamma \mathrm{d}} \left(1 + \Gamma_R\, e^{-2\gamma \mathrm{d}}\right)$$

$$I(\mathrm{d}) = \frac{V^{+} e^{\gamma \mathrm{d}}}{Z_0} \left(1 - \Gamma_R e^{-2\gamma \mathrm{d}}\right)$$

At each line location we define a Generalized Reflection Coefficient

$$\Gamma(\mathrm{d}) = \Gamma_R\, e^{-2j\beta \mathrm{d}}$$

$$\Gamma(\mathrm{d}) = \Gamma_R\, e^{-2\gamma \mathrm{d}}$$

and the line equations become

$$V(\mathrm{d}) = V^{+} e^{j\beta \mathrm{d}} \left(1 + \Gamma(\mathrm{d})\right)$$

$$I(\mathrm{d}) = \frac{V^{+} e^{j\beta \mathrm{d}}}{Z_0} \left(1 - \Gamma(\mathrm{d})\right)$$

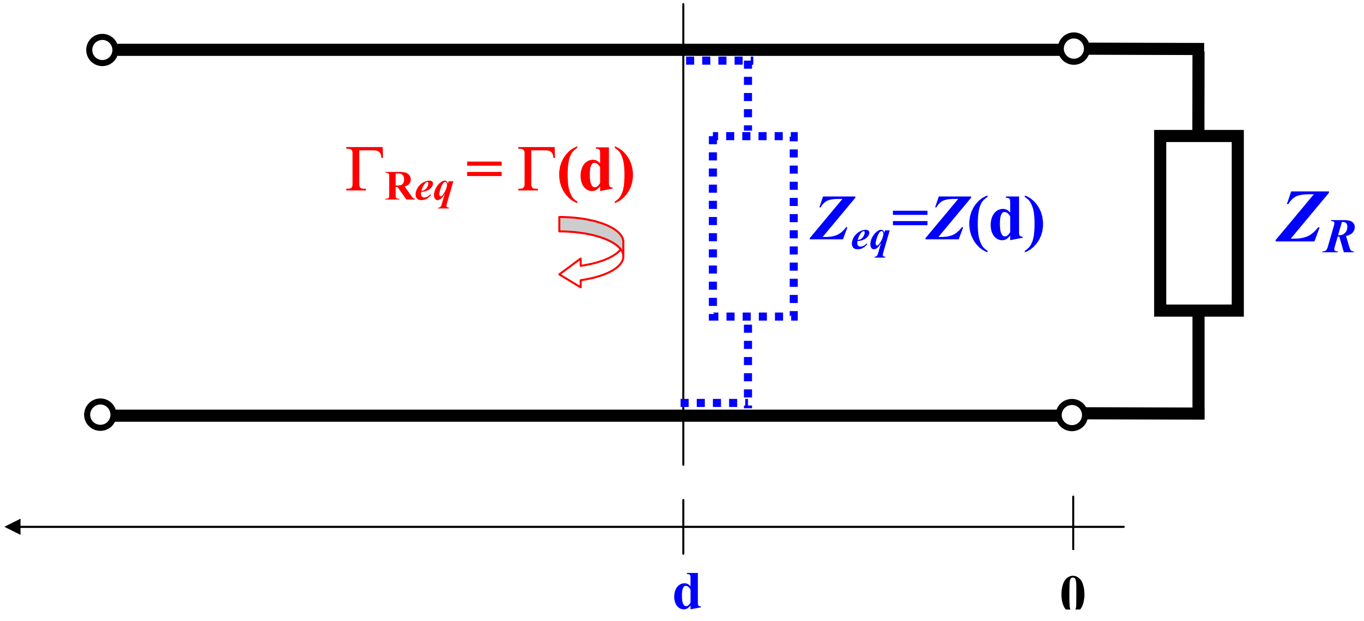
$$V(\mathrm{d}) = V^{+} e^{\gamma \mathrm{d}} \left(1 + \Gamma(\mathrm{d})\right)$$

$$I(\mathrm{d}) = \frac{V^{+} e^{\gamma \mathrm{d}}}{Z_0} \left(1 - \Gamma(\mathrm{d})\right)$$

We define the line impedance as

$$Z(\mathrm{d}) = \frac{V(\mathrm{d})}{I(\mathrm{d})} = Z_0 \frac{1+\Gamma(\mathrm{d})}{1-\Gamma(\mathrm{d})}$$

A simple circuit diagram can illustrate the significance of line impedance and generalized reflection coefficient:



If you imagine to cut the line at location **d**, the input impedance of the portion of line terminated by the load is the same as the line impedance at that location "before the cut". The behavior of the line on the left of location **d** is the same if an equivalent impedance with value ***Z*(d)** replaces the cut out portion. The reflection coefficient of the new load is equal to **Γ(d)**

$$\Gamma_{Req} = \Gamma(d) = \frac{Z_{Req} - Z_0}{Z_{Req} + Z_0}$$

If the total length of the line is **L**, the input impedance is obtained from the formula for the line impedance as

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V(\mathrm{L})}{I(\mathrm{L})} = Z_0 \frac{1+\Gamma(\mathrm{L})}{1-\Gamma(\mathrm{L})}$$

The input impedance is the equivalent impedance representing the entire line terminated by the load.

The characteristic impedance of the low-loss line is a real quantity for all practical purposes and it is approximately the same as in a corresponding loss-less line

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}}$$

and the phase velocity associated to the wave propagation is

$$v_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}$$

BUT NOTE:

In the case of the low-loss line, the equations for voltage and current retain the same form obtained for general lossy lines.

Again, we obtain the loss-less transmission line if we assume

$$R = 0 \qquad\qquad G = 0$$

This is often acceptable in relatively short transmission lines, where the overall attenuation is small.

As shown earlier, the characteristic impedance in a loss-less line is exactly real

$$Z_0 = \sqrt{\frac{L}{C}}$$

while the propagation constant has no attenuation term

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta$$

The loss-less line does not dissipate power, because $\alpha = 0$.

For all cases, the line impedance was defined as

$$Z(\mathrm{d}) = \frac{V(\mathrm{d})}{I(\mathrm{d})} = Z_0 \frac{1+\Gamma(\mathrm{d})}{1-\Gamma(\mathrm{d})}$$

By including the appropriate generalized reflection coefficient, we can derive alternative expressions of the line impedance:

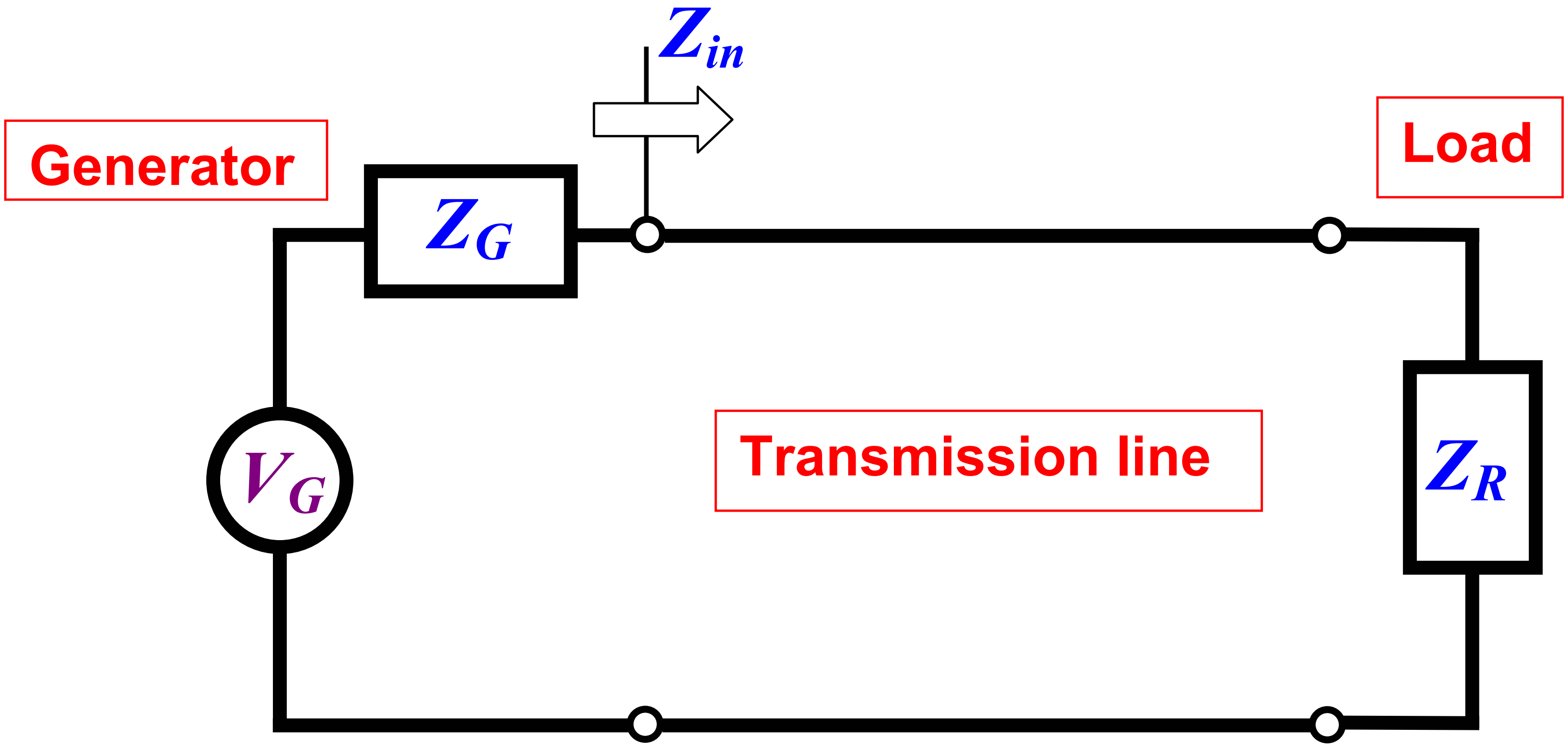
A) Loss-less line

$$Z(\mathrm{d}) = Z_0 \frac{1+\Gamma_R e^{-2j\beta \mathrm{d}}}{1-\Gamma_R e^{-2j\beta \mathrm{d}}} = Z_0 \frac{Z_R + jZ_0 \tan(\beta\,\mathrm{d})}{jZ_R \tan(\beta\,\mathrm{d}) + Z_0}$$

B) Lossy line (including low-loss)

$$Z(\mathrm{d}) = Z_0 \frac{1+\Gamma_R e^{-2\gamma \mathrm{d}}}{1-\Gamma_R e^{-2\gamma \mathrm{d}}} = Z_0 \frac{Z_R + Z_0 \tanh(\gamma\,\mathrm{d})}{Z_R \tanh(\gamma\,\mathrm{d}) + Z_0}$$

4. Maximal power transmission



$Z_G = Z_{in}^*$ **for maximum power transfer**

The characteristic impedance of the loss-less line is real and we can express the power flow, anywhere on the line, as

$$\langle P(\mathrm{d},t)\rangle = \frac{1}{2}\,\mathrm{Re}\{\,V(\mathrm{d})\,I^{*}(\mathrm{d})\,\}$$

$$= \frac{1}{2}\,\mathrm{Re}\left\{V^{+}e^{j\beta\,\mathrm{d}}\left(1+\Gamma_{R}e^{-j2\beta\,\mathrm{d}}\right)\right.$$

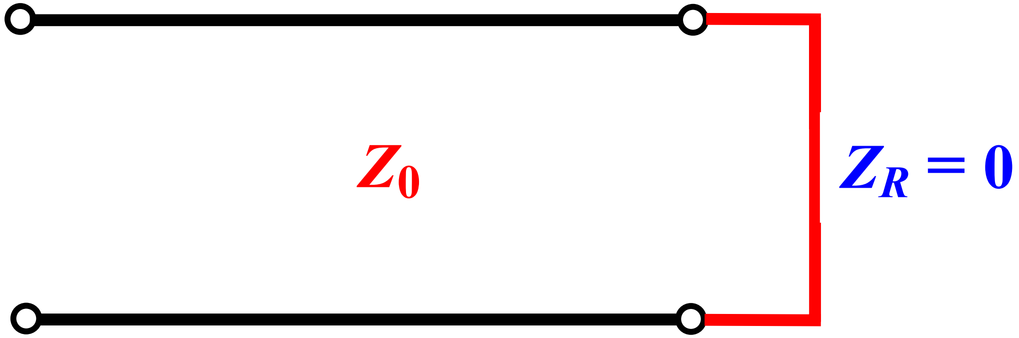
$$\left.\frac{1}{Z_0}(V^{+})^{*}e^{-j\beta\,\mathrm{d}}\left(1-\Gamma_{R}e^{-j2\beta\,\mathrm{d}}\right)^{*}\right\}$$

$$= \underbrace{\frac{1}{2Z_0}\left|V^{+}\right|^{2}}_{\boxed{\text{Incident wave}}} - \underbrace{\frac{1}{2Z_0}\left|V^{+}\right|^{2}\left|\Gamma_{R}\right|^{2}}_{\boxed{\text{Reflected wave}}}$$

This result is valid for any location, including the input and the load, since the transmission line does not absorb any power.

Particular cases

$\boxed{Z_R \to 0}$ **(SHORT CIRCUIT)**



The load boundary condition due to the short circuit is $V(0) = 0$

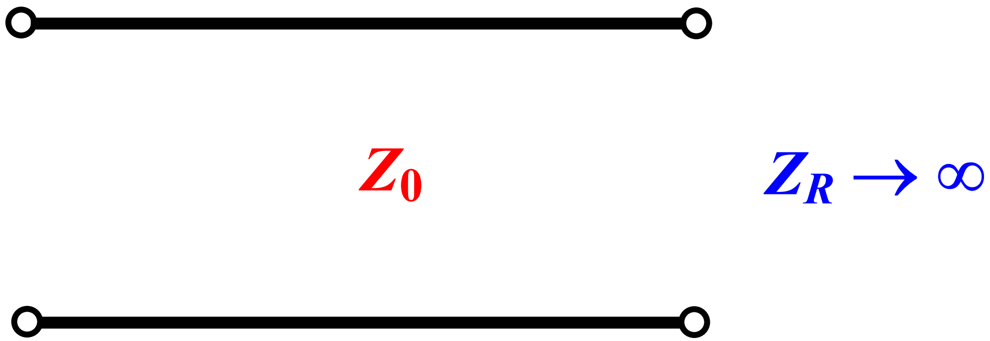
$$\Rightarrow V(\mathrm{d}=0) = V^+ e^{j\beta 0}(1+\Gamma_R\, e^{-j2\beta 0})$$

$$= V^+(1+\Gamma_R) = 0$$

$$\Rightarrow \quad \Gamma_R = -1$$

Particular cases

$\boxed{Z_R \rightarrow \infty \quad \textbf{(OPEN CIRCUIT)}}$



The load **boundary condition** **due to the open circuit is** $I\,(0) = 0$

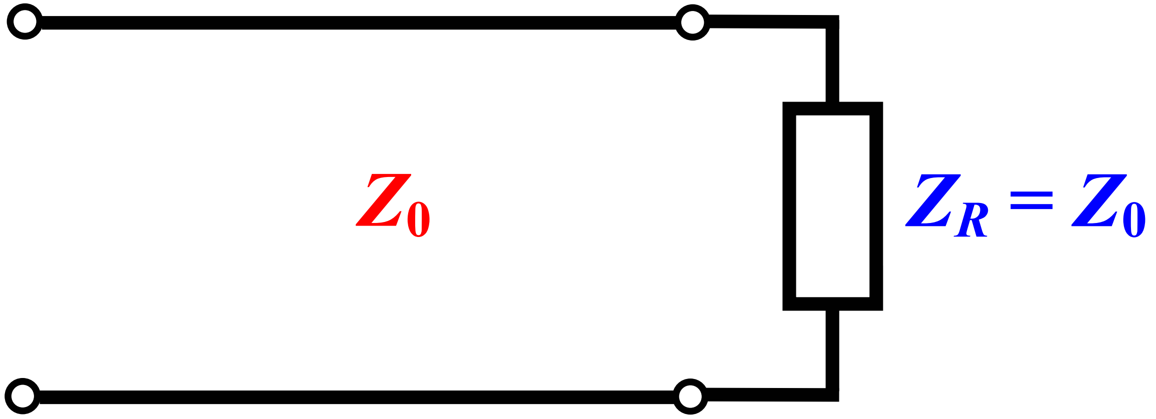
$$\Rightarrow I(\mathrm{d}=0) = \frac{V^+}{Z_0} e^{j\beta 0}(1-\Gamma_R\, e^{-j2\beta 0})$$

$$= \frac{V^+}{Z_0}(1-\Gamma_R) = 0$$

$$\Rightarrow \quad \Gamma_R = 1$$

Particular cases

$Z_R = Z_0$ **(MATCHED LOAD)**



The reflection coefficient for a matched load is

$$\Gamma_R = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

no reflection!

Particular cases

Short circuited transmission line – Fixed frequency

L

$\mathrm{L} = 0$	$Z_{in} = 0$	short circuit
$0 < \mathrm{L} < \frac{\lambda}{4}$	$\mathrm{Im}\{Z_{in}\} > 0$	inductance
$\mathrm{L} = \frac{\lambda}{4}$	$Z_{in} \to \infty$	open circuit
$\frac{\lambda}{4} < \mathrm{L} < \frac{\lambda}{2}$	$\mathrm{Im}\{Z_{in}\} < 0$	capacitance
$\mathrm{L} = \frac{\lambda}{2}$	$Z_{in} = 0$	short circuit
$\frac{\lambda}{2} < \mathrm{L} < \frac{3\lambda}{4}$	$\mathrm{Im}\{Z_{in}\} > 0$	inductance
$\mathrm{L} = \frac{3\lambda}{4}$	$Z_{in} \to \infty$	open circuit
$\frac{3\lambda}{4} < \mathrm{L} < \lambda$	$\mathrm{Im}\{Z_{in}\} < 0$	capacitance

…

Particular cases

Open circuit transmission line – Fixed frequency

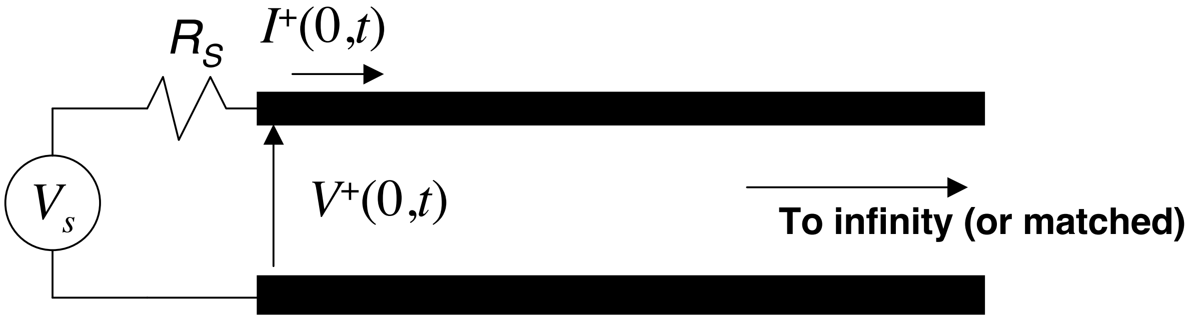
L

$\mathrm{L} = 0$	$Z_{in} \to \infty$	open circuit
$0 < \mathrm{L} < \frac{\lambda}{4}$	$\mathrm{Im}\{Z_{in}\} < 0$	capacitance
$\mathrm{L} = \frac{\lambda}{4}$	$Z_{in} = 0$	short circuit
$\frac{\lambda}{4} < \mathrm{L} < \frac{\lambda}{2}$	$\mathrm{Im}\{Z_{in}\} > 0$	inductance
$\mathrm{L} = \frac{\lambda}{2}$	$Z_{in} \to \infty$	open circuit
$\frac{\lambda}{2} < \mathrm{L} < \frac{3\lambda}{4}$	$\mathrm{Im}\{Z_{in}\} < 0$	capacitance
$\mathrm{L} = \frac{3\lambda}{4}$	$Z_{in} = 0$	short circuit
$\frac{3\lambda}{4} < \mathrm{L} < \lambda$	$\mathrm{Im}\{Z_{in}\} > 0$	inductance

…

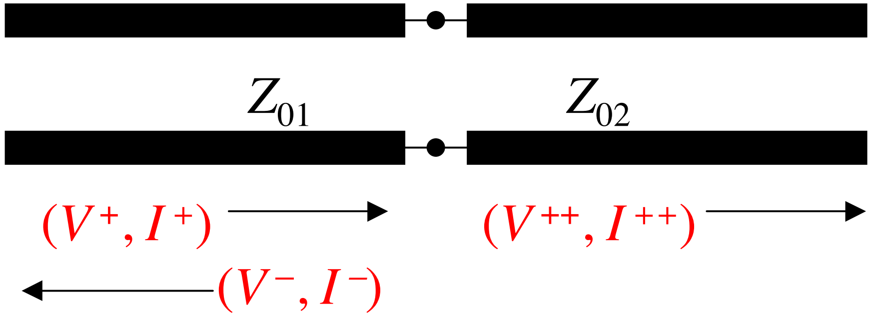
5. Transient propagation

The characteristic impedance dictates the amplitude of the voltage waveform launched on the line



$$\left.\begin{aligned} V_S(t) &= V^+(0,t) + I^+(0,t)R_S \\ V^+(0,t) &= Z_0 I^+(0,t) \end{aligned}\right\} \Rightarrow \boxed{V^+(0,t) = V_S(t)\frac{Z_0}{Z_0 + R_S}}$$

Discontinuities in the characteristic impedance of a transmission line give rise to reflections



At the junction it is:

$$V_1 = V^+ + V^- = V_2 = V^{++}$$

$$I_1 = \frac{1}{Z_{01}}\left(V^+ - V^-\right) = I_2 = \frac{V^{++}}{Z_{02}}$$

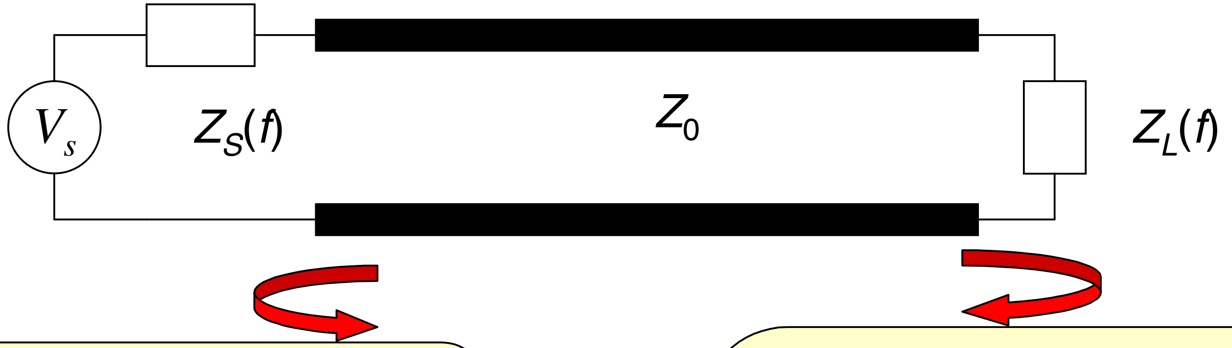
$$\left.\begin{aligned} V^- &= \Gamma V^+ \\ I^- &= -\frac{V^-}{Z_{01}} \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} V^{++} &= T V^+ \\ I^{++} &= \frac{V^{++}}{Z_{02}} \end{aligned}\right\}$$

Reflection Coefficient: $\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$

Transmission Coefficient: $T = \frac{2Z_{02}}{Z_{02} + Z_{01}}$

Maintaining a fairly constant value of the characteristic impedance along an interconnect path is essential for reflection suppression.

Source and load impedances impact transmission line performance of the interconnect



Source reflection coefficient

$$\Gamma_S(f) = \frac{Z_S(f) - Z_0}{Z_S(f) + Z_0}$$

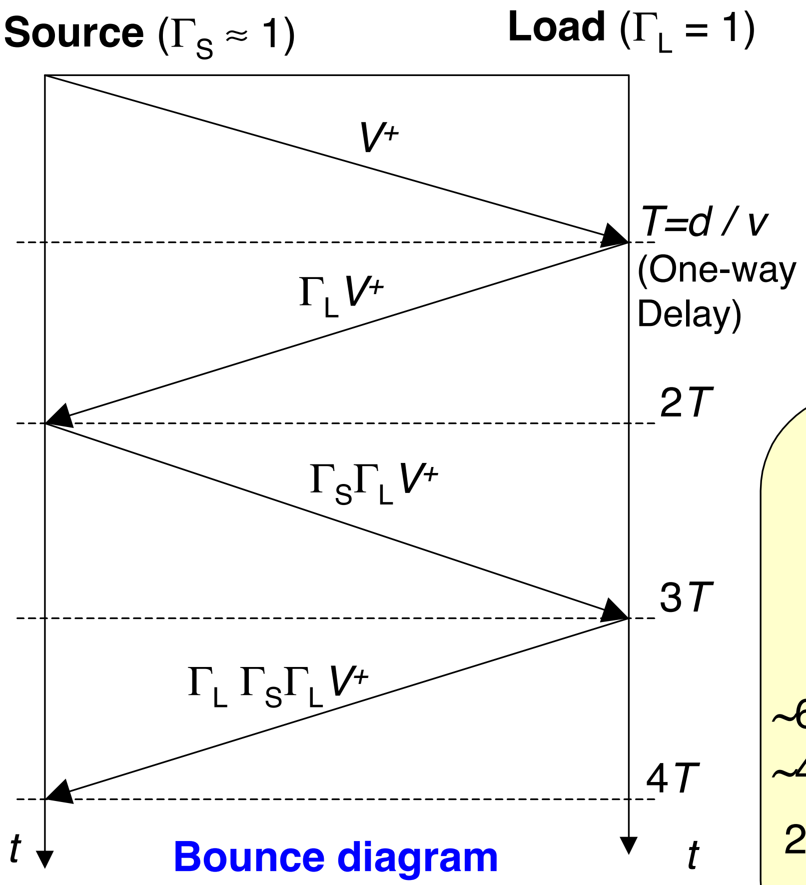
Load reflection coefficient:

$$\Gamma_L(f) = \frac{Z_L(f) - Z_0}{Z_L(f) + Z_0}$$

Load transmission coefficient:

$$T_L(f) = 1 + \Gamma_L(f) = \frac{2Z_L(f)}{Z_L(f) + Z_0}$$

Example: Unterminated interconnect ($Z_L=\infty$) driven by high source impedance driver with $Z_S>>Z_0$ (e.g. unbuffered CMOS)

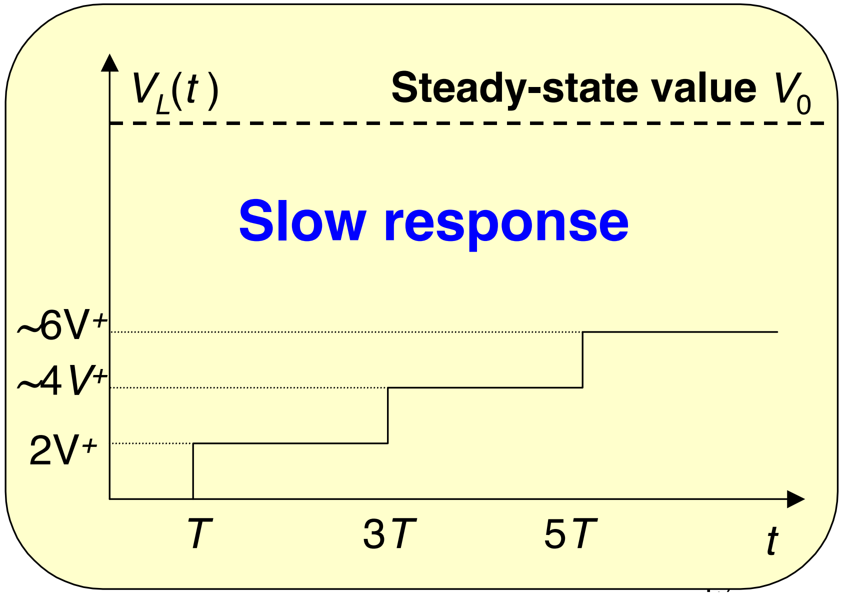


Bounce diagram

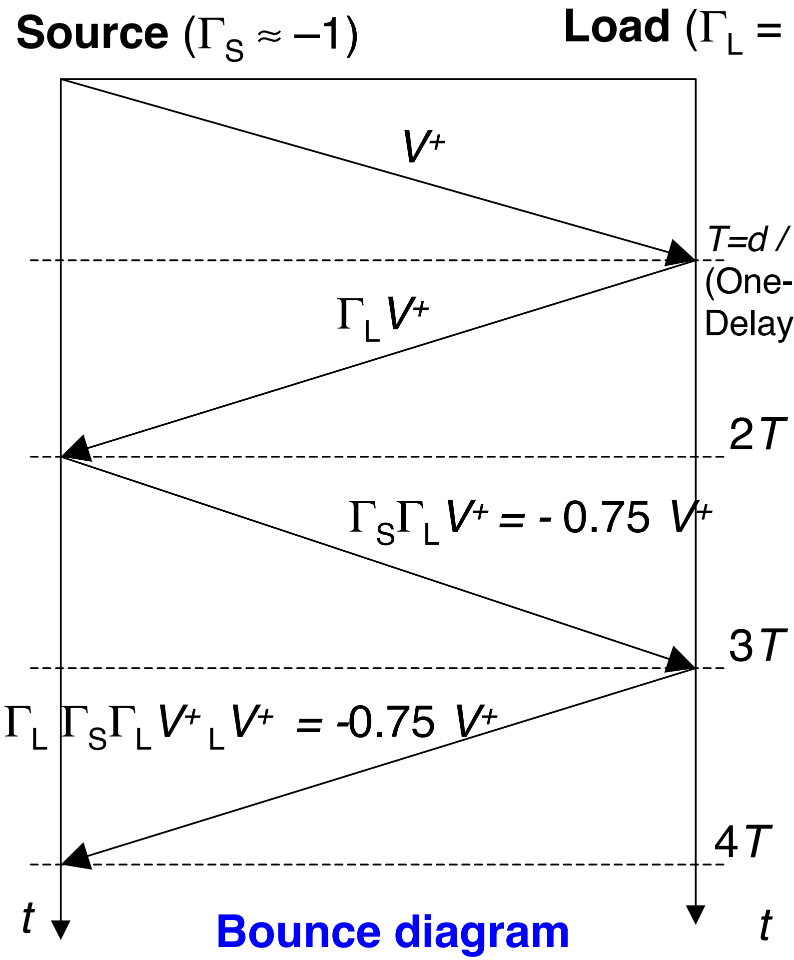
Excitation: Step Pulse of amplitude V_0

$$V^+ = V_0 \frac{Z_0}{Z_S + Z_0} << V_0, \; \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \approx 1$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1, \quad T_L = 2$$



Example: Unterminated interconnect (Z_L=∞) driven by low source impedance driver with Z_S<Z_0 (e.g. ECL or strong TTL)



Bounce diagram

Excitation: Step Pulse of amplitude V_0; $Z_0 = 7Z_S$

$$V^+ = V_0 \frac{Z_0}{Z_S + Z_0} \approx \frac{7}{8} V_0,\ \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = -0.75$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1, \quad T_L = 2$$