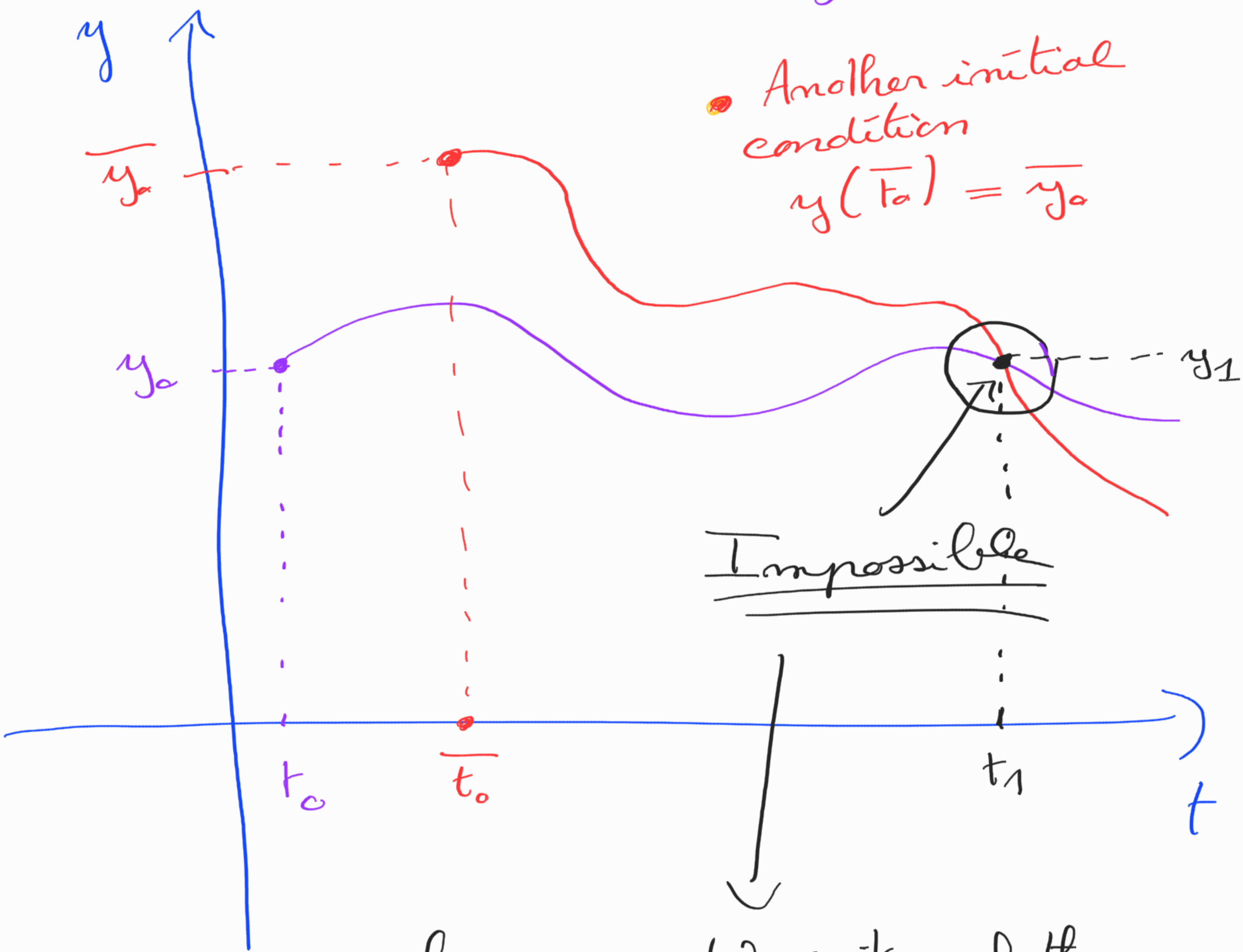
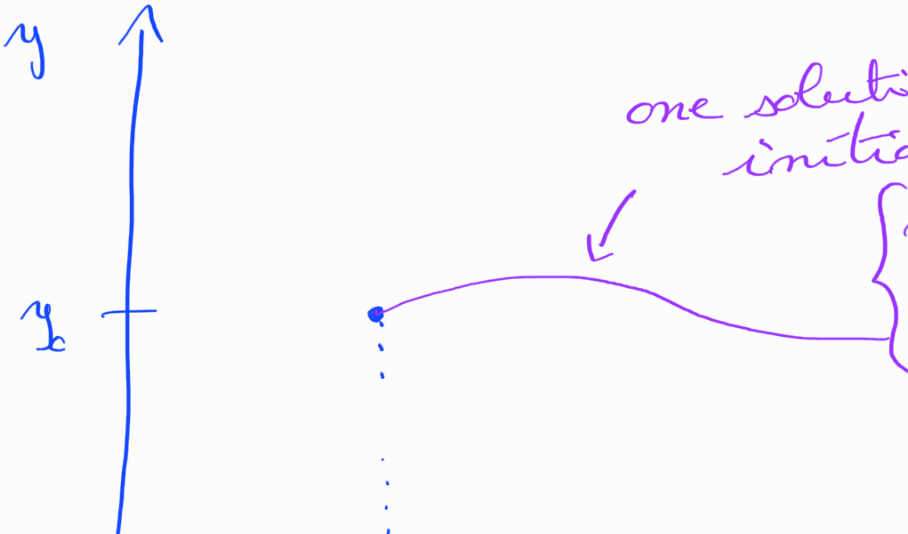
$$y'(t) = f(y, t)$$

- Initial condition $y(t_0) = y_0$
- Another initial condition $y(\bar{t}_0) = \bar{y}_0$



because: Unicity of the solution for one given initial condition (Cauchy Theorem)



one solution for one initial condition

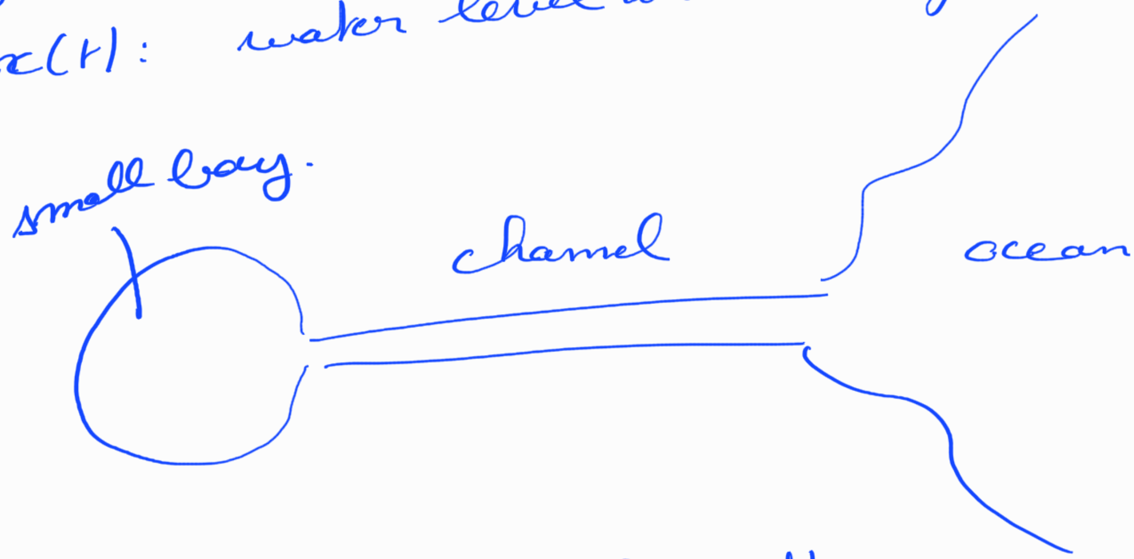
$$\begin{cases} y(t_0) = y_0 \\ y'(t) = f(y, t) \end{cases}$$

t_a $\qquad t$

Ex 2 : Small bay connected to the ocean

$y(t)$: heigh of ocean

$x(t)$: water level in the bay.



small time intervall : dt

variation of water level in bay : dx

1) dx : proportional to $(y(t) - x(t))$
also proportional to dt

Assumption

$$\boxed{dx = k\,(y(t) - x(t))\,dt}$$

$$\frac{dx}{dt} = k\,(y(t) - x(t))$$

$\downarrow dt \to 0$

$$x'(t) = k\,(y(t) - x(t))$$

$x'(t) = k(y$

2) tide every 4π hours.

ocean height : $y(t) = \cos(\omega t)$

$y(t)$ is 4π – periodic

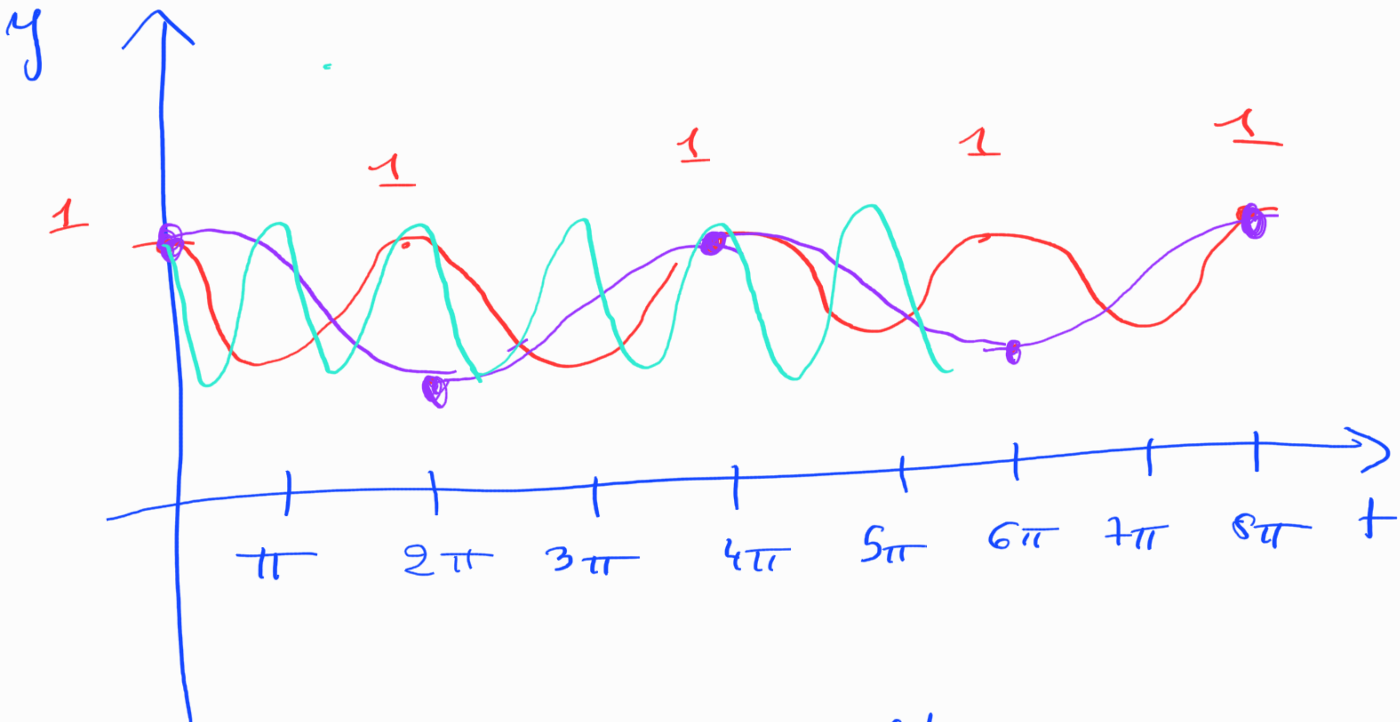
when t varies between 0 and 4π

we must have $\cos(\omega t)$ having one complete period.

+ cos is 2π periodic.

$\Rightarrow$ ωt varies between 0 and 2π when t varies between 0 and 4π

$\Rightarrow \omega = 1/2$



3) $$\int e^{kt}\cos(\omega t)\,dt = \frac{1}{k^2+\omega^2}\, e^{kt}\left(k\cos(\omega t) + \omega\sin(\omega t)\right) + C$$

$g(t)$

$$g'(t) = \frac{1}{k^2+\omega^2}\Big[k e^{kt}\big(k\cos(\omega t)+\omega\sin(\omega t)\big) + e^{kt}\big(-k\omega\sin(\omega t)+\omega^2\cos(\omega t)\big)\Big]$$

$$= \frac{e^{kt}}{k^2+\omega^2}\Big[k^2\cos(\omega t) + k\omega\cancel{\sin}(\omega t) - k\omega\cancel{\sin}(\omega t) + \omega^2\cos(\omega t)\Big]$$

$$= \frac{e^{kt}}{\cancel{k^2+\omega^2}}\big[\cancel{k^2+\omega^2}\big]\cos(\omega t)$$

4) $\begin{cases} x'(t) = k\,(y(t) - x(t)) \\ y(t) = \cos(\omega t) = \cos\left(\frac{t}{2}\right) \end{cases}$

$$\Rightarrow \quad x'(t) + k\,x(t) = k\cos(\omega t)$$

We look for u such that:

$$u x' + k u x = (ux)'$$

because it satisfies also $\underbrace{k u x = u' x}$

$$\Downarrow$$

$$k u(t) = u'(t)$$

$$\Rightarrow u(t) = C\, e^{kt}$$

We multiply the differential eq with u:

$$x'(t)u(t) + k\,x(t)u(t) = k\cos(\omega t)\,u$$

$$\underbrace{Ce^{kt}x'(t) + kCe^{kt}x(t)}_{} = k\cos(\omega t)e^{kt}C$$

$$\left(Ce^{kt}x(t)\right)' = Cke^{kt}\cos(\omega t)$$

$$\cancel{C}e^{kt}x(t) = \int Cke^{kt}\cos(\omega t)$$

$$= \cancel{C}k\underbrace{\int e^{kt}\cos(\omega t)}_{\frac{1}{k^2+\omega^2}e^{kt}\left(k\cos(\omega t)+\omega\sin(\omega t)\right)+C_1}$$

$$x(t) = \cancel{e^{-kt}}\, k\left[\frac{1}{k^2+\omega^2}\cancel{e^{kt}}\left[k\cos(\omega t)+\omega\sin(\omega t)\right] + C_1\right]$$

$$x(t) = \frac{k}{k^2+\omega^2}\left[k\cos(\omega t)+\omega\sin(\omega t)\right] + kC_1e^{-kt}$$

$C_1 = 0$
otherwise the term tends to $+\infty$ when $t \to +\infty$ or $-\infty$
which is unrealistic

Next Week: moodle course on epidemiology.

$\rightarrow$ explanations on next course.