Topic 1:
Method to synthesize analog filters

Aims:

The aim of this TA is to:

- be able to **synthesize an analog filter** from a given template,
- be able to calculate the elements of first- and second-order filters that satisfy the **characteristic angular frequency and quality factor** values imposed by specifications,
- understand the advantages of active analog filters based on OP-AMPs but also **their limitations**.

Prerequisites:

S1 and S2 courses on Circuits and Electronics. Bode diagram. Transfer function. Complex numbers. Logarithms.

I- Method to synthesize a “Butterworth” active low-pass filter

I-1. Theoretical study

I-1-a. Calculating order and characteristic frequency

Our aim is to synthesize a “Butterworth” analog low-pass filter type using the template shown in Figure 1. Other types of filter (Bessel, Chebyshev, Cauer, etc.) are too difficult to synthesize analytically and require the use of a computer. In order to study their properties and compare their performances, these other types of filter, especially Bessel and Chebyshev, will be dealt with in the PA.

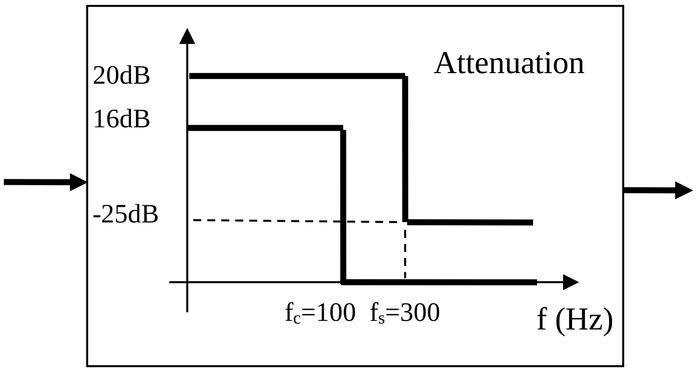

Figure 1: *Template for low-pass filter and digital values in specifications*

Frequencies f_c and f_s refer to the end of the **pass-band** and the start of the **stop-band** respectively.

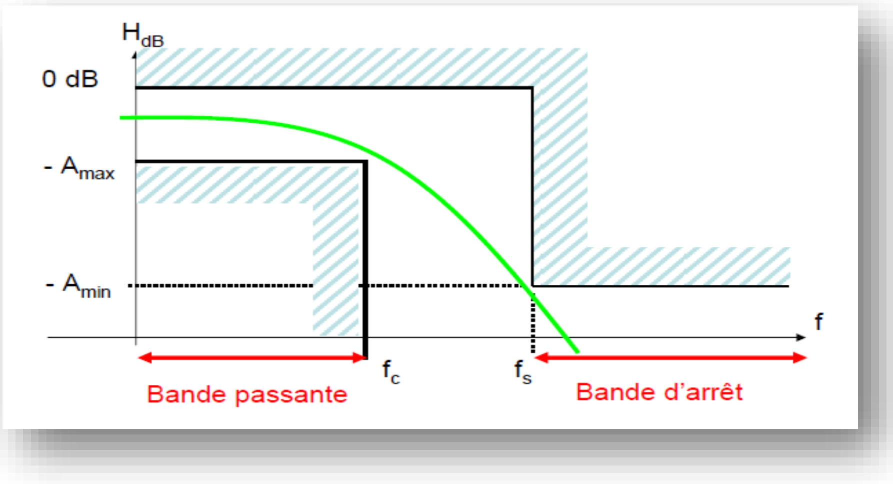

Figure 2: *Quantities associated with the normalized filter template*

NB: Quantities A_{max} and A_{min} correspond to maximum acceptable pass-band ripple and minimal acceptable stop-band attenuation respectively.

Question 1: The filter is produced by cascading together a low-pass filter and an amplifier with a gain of 10. Complete the digital values in the template of the low-pass filter in Figure 3.

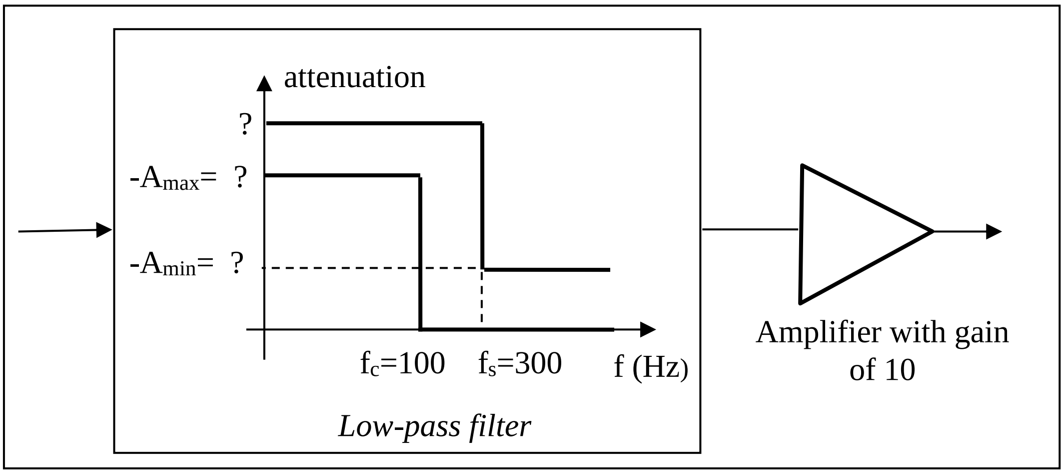

Figure 3: *Filter produced by cascading together a low-pass filter and an amplifier with a gain of 10.*

Question 2: The gain module of a low-pass Butterworth filter of order n and cutoff angular frequency ω_0 at -3dB is written:

$$|H(j\omega)| = \frac{1}{\left(1+\left[\frac{\omega}{\omega_0}\right]^{2n}\right)^{1/2}} \qquad (1)$$

- What values is $|H(j\omega)|$ tending towards when $\omega \to 0$ and $\omega \to \infty$ and what is the value of $|H(j\omega)|$ for the specific angular frequency $\omega = \omega_0$ also called the characteristic angular frequency?
- Explain why ω_0 corresponds to the cutoff angular frequency value at -3dB **(attention: this is not the case for all filters!).**

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Question 3: From the completed template in Figure 3 and equation (1), we obtain **2 equations with 2 unknowns** for the two specific points f_C and f_S. Derive the expression for **order n** and **angular frequency** ω_0. Start by calculating n, then select the integer just above this value and extract ω_0.

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I-1-b. Synthesizing the filter

Question 4: If your calculations are correct, you should find that the low-pass filter in Figure 2 represents a cascading together of two second-order filters and one first-order filter. Using the table in Appendix 1:

- for each of the second-order filters, give the value of the overvoltage coefficient Q and the characteristic angular frequency ω_0,
- for the first-order filter, give the value of the characteristic angular frequency ω_0.

	value of Q	**value of ω_0**
first second-order filter		
second second-order filter		
first-order filter		

<u>NB</u>: Remember that the transfer functions $H(p)$ (Laplace transform) for the first-order and second-order filters respectively are written in the following forms:

$$H(p) = \frac{\omega_0}{p + \omega_0} \qquad (2)$$

$$H(p) = \frac{\omega_0^2}{p^2 + p\dfrac{\omega_0}{Q} + \omega_0^2} \qquad (3)$$

Question 5: The first-order filter is made according to the diagram in Figure 4. The OP-AMP is assumed to be ideal.

- Check that the filter transfer function $H(p) = \dfrac{V_S(p)}{V_e(p)}$ is similar to that in equation 2 above.
- What is the advantage of putting in an OP-AMP since the gain is the same without an OP-AMP?

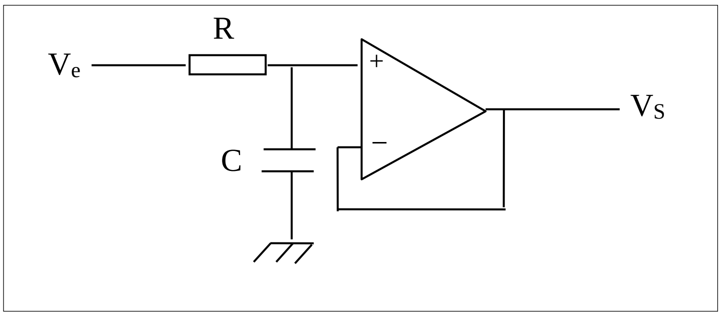

Figure 4: *First-order low-pass filter*

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Question 6: Assume $R = 10k\Omega$, calculate the value of C.

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The filter in Figure 1 requires a gain-of-10 amplifier, this gain can be included in the previous filter, as shown in the diagram in Figure 5.

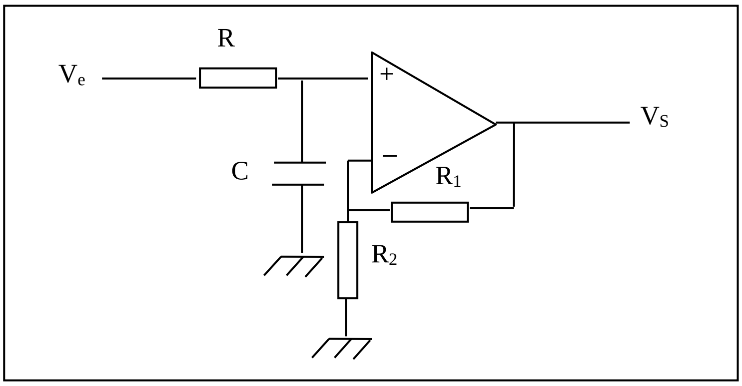

Figure 5: *Low-pass filter with gain for* $f \to 0$

Question 7:

- Show that the transfer function of this filter is written in the form:

$$H(p) = A_{LP}\,\frac{\omega_0}{p+\omega_0} \qquad (4)$$

where A_{LP} is the gain at zero frequency.

- Give the expression of A_{LP} as a function of R_1 and R_2. Choose $R_2 = 1\text{k}\Omega$. Give the value R_1 in order to obtain $A_{LP} = 10$.

The two second-order filters are made from active cells known as "Sallen-Key" (see diagram in Figure 6). The filter transfer function $H(p) = \dfrac{V_S(p)}{V_e(p)}$ shown in Figure 6 is given by equation 3. To obtain this, write the equations at nodes A and B.

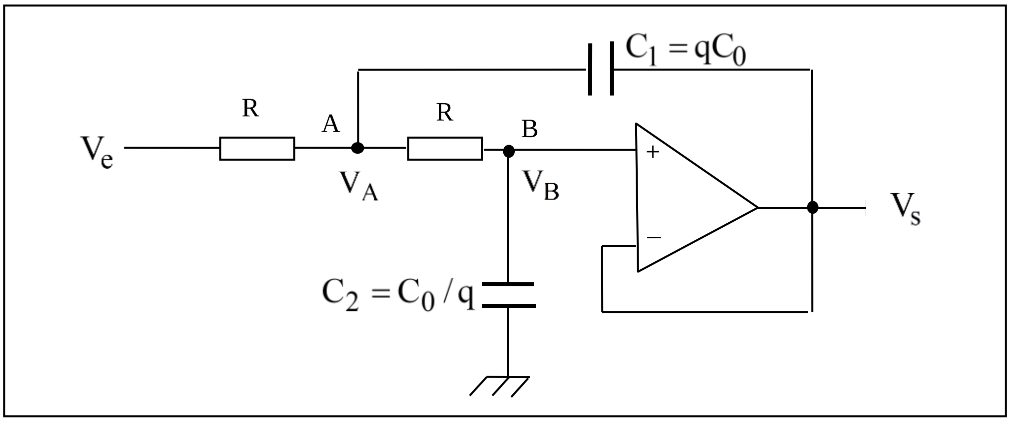

Figure 6: *Sallen-Key active cell to make a second-order filter*

Question 8: **As the OP-AMP is assumed ideal**, what approximately is the potential V_B at point B in relation to ground potential? Then write the equations at nodes A and B as a function of V_e, V_s, V_A, R and the functional admittances qC_0p and $\frac{C_0}{q}p$.

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Question 9: To obtain $H(p) = \frac{V_s(p)}{V_e(p)}$, extract the potential V_A from one of the 2 equations then replace it in the other equation. This should give an equation identical to equation 3. Identify the expressions of Q as a function of q then of ω_0 as a function of R and C_0.

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Question 10: $R = 10k\Omega$ is given. Calculate capacitances C_1 and C_2 to place on the circuit for each filter.

	value of Q	**value of** ω_0	**R**	C_1	C_2
first second-order filter			$10k\Omega$		
second second-order filter			$10k\Omega$		

Question 11:

- Give the full diagram for the filter.
- Can the position of the filter blocks be reversed in the final circuit? Justify your answer.

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Question 12:

- Write the expression for the complex gains of each filter for frequencies very much higher than the cutoff frequency f_0.
- Then write the full gain of the filter in dB, i.e. $20\log_{10}|H(j\omega)|$. From this determine the attenuation slope (in dB/decade) for frequencies $f >> f_0$.

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Question 13: What are the phase differences between output and input signals for $f \to 0$ and $f \to \infty$? Justify your answer.

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I-2. Practical simulation

I-2-a. Presentation of Tina-TI software

Tina-TI (available as a free download on the Texas Instruments website www.ti.com) is an electronic circuit simulation software package. It includes a SPICE model library of OP-AMPs, instrumentation amplifiers, comparators, voltage regulators, etc., produced by Texas Instruments. After entering the circuit, different analyses are possible (Figures 7 and 8):

- DC analysis (analysis of magnitudes in continuous regime)
- AC analysis (analysis of magnitudes in variable regime)
- Transient analysis (analysis of magnitudes in transient regime)
- Fourier analysis (frequency domain analysis)
- Noise analysis

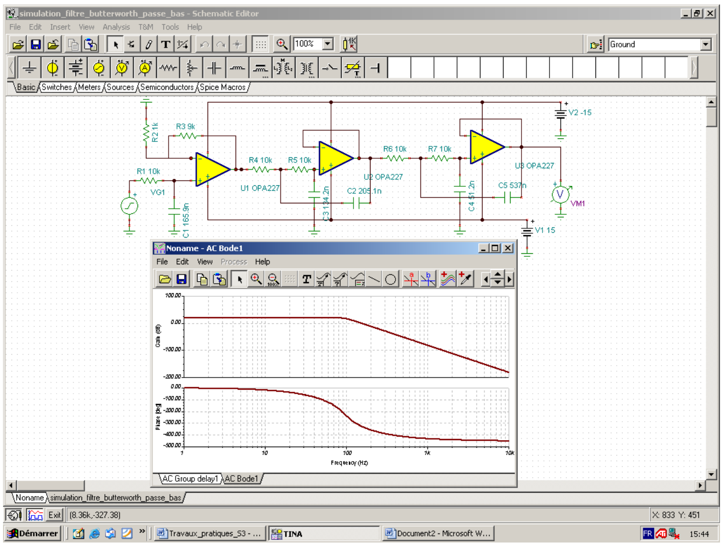


Figure 7: *Tina-TI software sample screen: circuit schematic and result of AC analysis simulation (Bode diagram)*

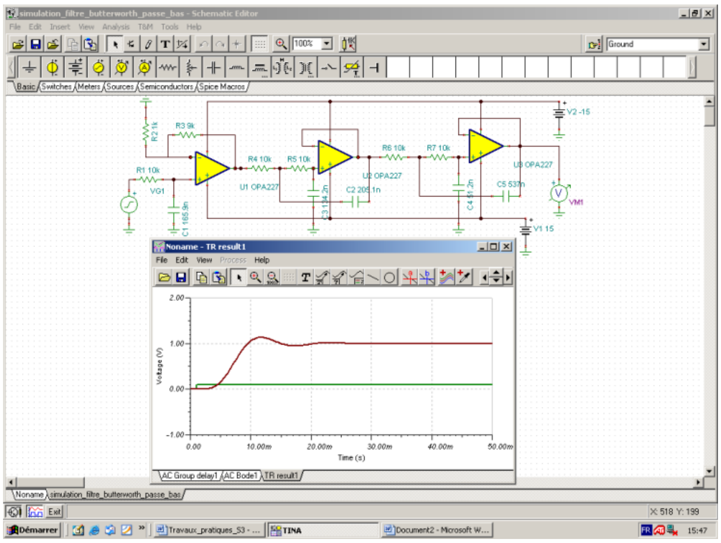


Figure 8: *Tina-TI software sample screen: circuit schematic and result of Transient Analysis simulation*

The software is fairly easy to use and user-friendly.

- To select a component, scroll down and click OK in the component selection window then drag it into position in the circuit workspace.
- To set the value of a component, double-click on it and fill in the required fields.
- Draw in wires between the components and move on to one of the analyses mentioned above.

NB: Tina-TI software only simulates active circuits. However, this is not a problem as you can simply add a follower type assembly, for example, in order to analyze a circuit with only passive components.

I-2-b. Studying the filter in harmonic regime

➢ **Enter** the circuit schematic of the Butterworth filter, adding the values calculated previously. Put the first-order filter in place first.

Procedure for schematic entry:

- Select OP-AMPs: Horizontal navigation bar → Spice Macros → Operational Amplifiers → OPA227
- Select passive components (Resistor, Capacitor, Ground, Battery): Horizontal navigation bar → Basic
- Select generator: Horizontal navigation bar → Sources → Voltage Generator
- Select measurement equipment: Horizontal navigation bar → Meters → Volt Meter

➢ **Study in harmonic regime**:

Procedure to obtain complex gain module and phase:

- Horizontal navigation bar → Analysis → AC Analysis → AC Transfer Characteristic
- Complete the fields in the window (Start Frequency, End Frequency, Number of points, Sweep type Logarithmic, Diagram Amplitude & Phase)

I-2-c. Applying the results

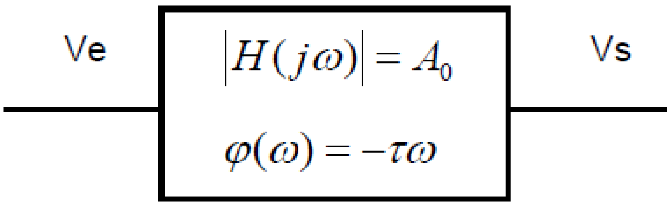
Question 14: Using the zoom function and cursors, determine:

- gain at low frequencies ($f \to 0$)
- cutoff frequency f_{-3dB}
- attenuation slope in dB/decade for $f \to \infty$
- phase shift for $f \to \infty$
- cutoff frequency f^1_{-3dB} of the first-order filter placed at the top of the filter, give the value of k in $f^1_{-3dB} = kf_{-3dB}$. Compare this to the theoretical value.
- Compare the values obtained from the simulation with those calculated previously.

I-2-d. Determining group delay

The purpose of a low-pass filter is to eliminate frequencies in the stop band and to allow frequencies in the pass-band to pass through. If the signal applied to a filter input includes several frequencies in its pass-band, it is clearly better for these frequencies to have the **same delay** so that the useful signal is simply delayed and is therefore not distorted (Figures 9 and 10).

Exemple : $v_e(t)$ = A1 $\cos(\omega_1 t)$+A2 $\cos(\omega_2 t)$+A3 $\cos(\omega_3 t)$ avec $\omega_0 >> \omega_{1,2,3}$



$$v_s(t) = A_0\,[A1\cos(\omega_1 t+\phi_1))+A2\cos(\omega_2 t+\phi_2)+A3\cos(\omega_3 t+\phi_3)]$$

$$v_s(t) = A_0\,[A1\cos(\omega_1 t-\tau\omega_1))+A2\cos(\omega_2 t-\tau\omega_2)+A3\cos(\omega_3 t-\tau\omega_3)]$$

$$v_s(t) = A_0\,[A1\cos(\omega_1\,(t-\tau)\,)+A2\cos(\omega_2\,(t-\tau)\,)+A3\cos(\omega_3\,(t-\tau)\,)]$$

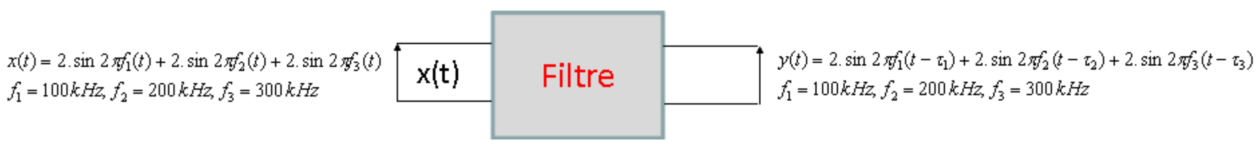
$$v_s(t) = A_0\,v_e(t-\tau)$$

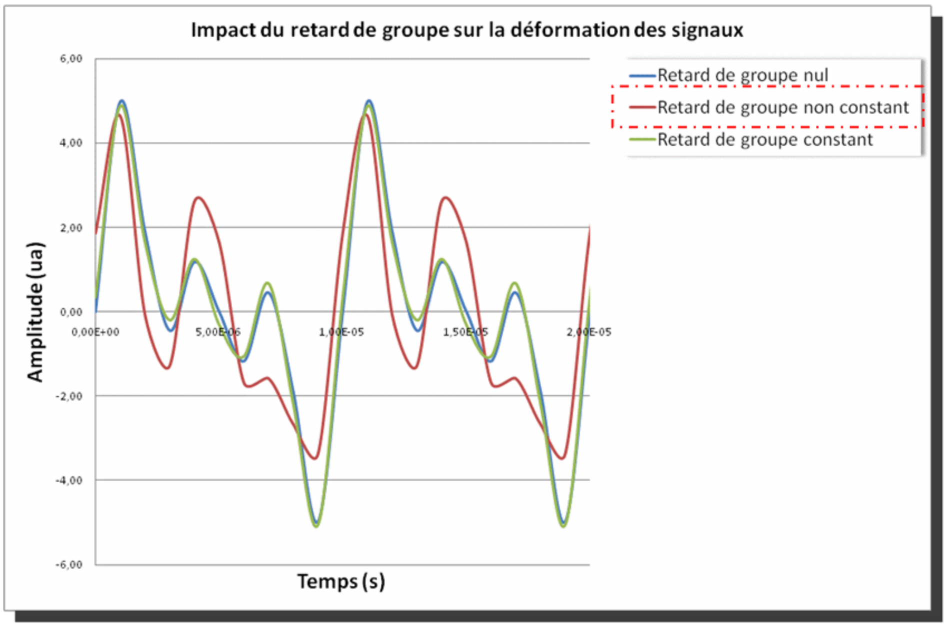
Figure 9: *Notion of group delay on a multi-frequency input signal*

- All frequencies pass through the filter in a time that is exactly equal.
- As you will see in the PA, **only the Bessel filter** has this capability, minimizing the distortion that a complex signal is subject to during a filter operation.

For a given angular frequency $\omega = \frac{2\pi}{T}$, the delay is $\frac{T}{2\pi}\varphi(\omega) = \frac{\varphi(\omega)}{\omega}$ where T is the time period and $\varphi(\omega)$ is the filter phase shift at angular frequency ω.

To obtain a constant delay, whatever the value of ω, then it must be the case that $\frac{\varphi(\omega)}{\omega} = C^{te}$, or phase $\varphi(\omega)$ must be proportional to the angular frequency, i.e. $\varphi(\omega) = C^{te}\omega$, then the delay would be equal to C^{te} for all angular frequencies. No analog filter is able to ensure a constant delay for all angular frequencies, but some filters have a more linear phase than others, and this is the case for **Bessel filters**, which we will study next.





Impact du retard de groupe sur la déformation des signaux=> Impact of group delay on signal distortion Retard de groupe nul=> Zero group delay Retard de groupe non constant=> Non-constant group delay Retard de groupe constant=> Constant group delay Amplitude=>Amplitude Temps=>Time

Figure 10: *Illustration of the influence of group delay on a multi-frequency input signal*

To define the linearity of the phase as a function of frequency, we introduce group delay $\tau(\omega) = -\dfrac{d\varphi}{d\omega}$, derived from the phase in relation to angular frequency. If the phase is proportional to the angular frequency, then τ is a constant (see Figure 11).

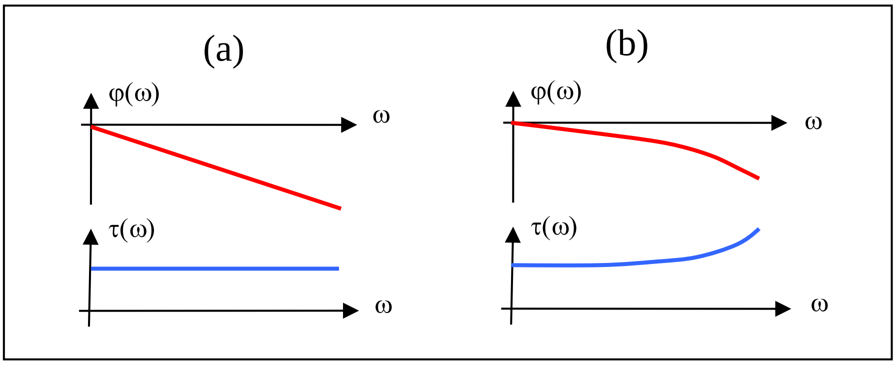


Figure 11: *(a) linear phase → constant group delay → non-distorted delayed signal, (b) non-linear phase → non-constant group delay → distorted signal*

To obtain the group delay, proceed as follows:

- Horizontal navigation bar → Analysis → AC Analysis → AC Transfer Characteristic: complete the fields in the window (Start Frequency, End Frequency, Number of points, Sweep type Logarithmic, Group Delay)

Question 15:

- Give the group delay for $f \to 0$
- Is the group delay constant in the pass-band?
- What is the maximum value of group delay in the pass-band?

I-2-e. Step response or step function response

Procedure to follow to obtain the step response:

- Double-click on generator →
  Signal Unit = Step, Amplitude = 100mV, Start of edge = 0s
- Horizontal navigation bar → Analysis → AC Analysis → Transient: complete the fields in the window (Start display, End display, Draw excitation)

Question 16:

- Give the settling time at 1%.
- What is the delay at 50% of the final value? You must check that the delay is of the same order of magnitude as the group delay measured previously.

Definition of "settling time" (Figure 12):
*The **settling time** of an amplifier or other output device is the time lapse from the application of an ideal instantaneous step input to the time at which the amplifier output has entered and remained within a specified error band, usually symmetrical around the final value.*

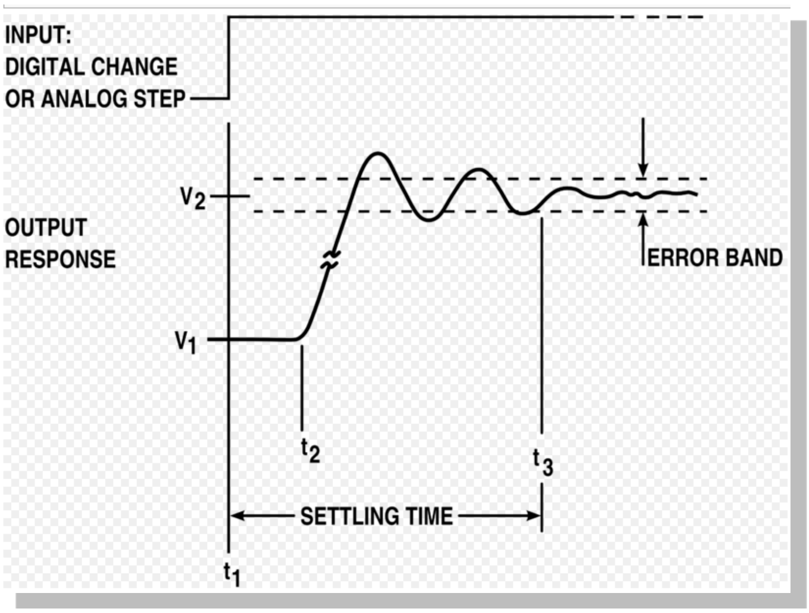


Figure 12: *Definition of "settling time"*

NB: Step response and frequency response are obviously linked (TF (derived from the step response) = frequency response). We observe that the step response of the Butterworth filter is "fairly" disturbed, which is directly linked with the fact that group delay varies in the pass-band. We shall see that the step response of the Bessel filter, on the other hand, is much less disturbed.

II- Effect of limitations of using OP-AMPs

II-1. Theoretical study – non-ideal OP-AMP

In the previous questions, **OP-AMPs were assumed to be ideal with an infinite open-loop gain**, i.e. an input-output relation: $V_s = A_d(V^+ - V^-)$ with $A_d \to \infty$.

In practice, the OP-AMP has its own transfer function:

$$V_s = \frac{A_d}{(1 + j\frac{\omega}{\omega_c})}(V^+ - V^-) \qquad (5)$$

where A_d is the static gain (f→0) in difference mode and ω_c is the cutoff angular frequency at -3dB in open loop.

Figure 13 shows the gain $\frac{V_s}{(V^+ - V^-)}$ of OP-AMP TL081 as a function of frequency.

Question 17: Use this figure to determine the values of A_d and f_C.

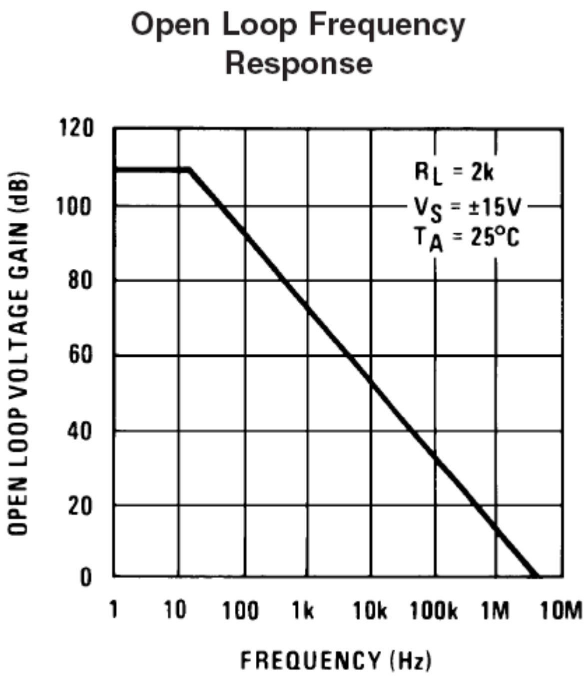


Figure 13: *Open loop gain of OP-AMP TL081*

Question 18: Determine the complex gain $H(j\omega) = \frac{Vs}{Ve}$ of the assembly in Figure 14.

To do this, write the voltage $(V^+ - V^-)$ and transfer it into the expression for OP-AMP open-loop gain.

If you assume $A_d >> (1+\frac{R_1}{R_2})$, then you should obtain an expression that takes the form:

$$H(j\omega) = \frac{Vs}{Ve} \approx \left(1+\frac{R_1}{R_2}\right)\frac{1}{1+j\frac{\omega}{\omega_c^{'}}} \quad \text{with } \omega_c^{'} \approx \omega_c \frac{A_d}{(1+\frac{R_1}{R_2})} \qquad (6)$$

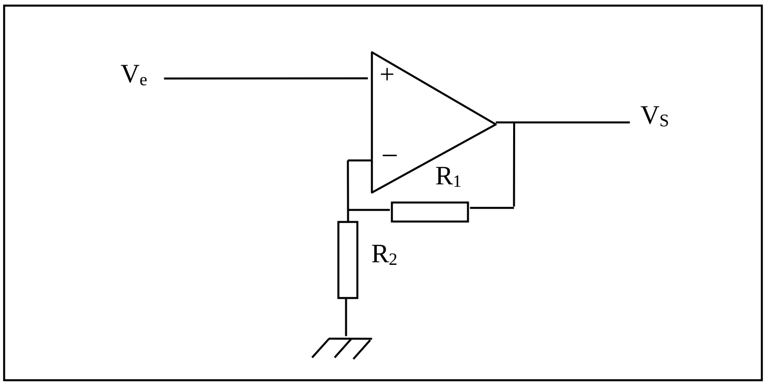


Figure 14: *Operational amplifier with limited pass-band*

Question 19:
What is the role played by this assembly if we take account of its limited pass-band?

Next, we want to make a filter according to the template in Figure 15. The filter is produced by cascading together a filter without static gain followed by an amplifier with gain of 10000, as shown in the general diagram in Figure 3.

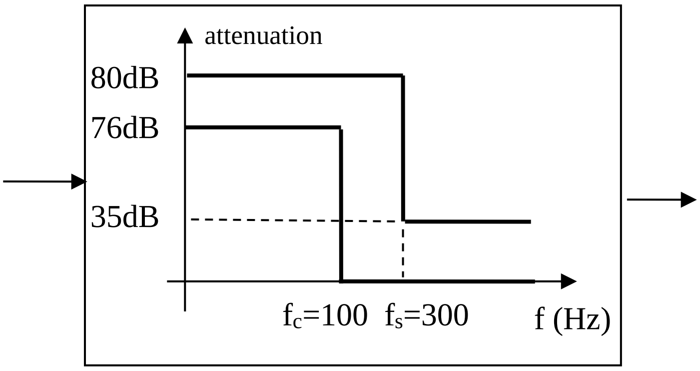


Figure 15: *Template of low-pass filter*

Question 20: The gain of 10000 is achieved with the assembly in Figure 15.

- Calculate the cutoff frequency of the assembly at -3dB.
- Ultimately, what order of filter will be used (4^{th}, 5^{th} or 6^{th})? Justify your answer. In this specific case, is this an advantage or a disadvantage?

Question 21:
The specifications are modified and require 100dB instead of 80dB, what happens?

To answer this question, we continue to use equation (6) to calculate $\omega_c^{'}$ even though the condition $A_d >> (1+\frac{R_1}{R_2})$ is no longer really satisfied.

II-2. Practical simulation

Modify your general diagram to produce a Butterworth filter that satisfies the template in Figure 16.

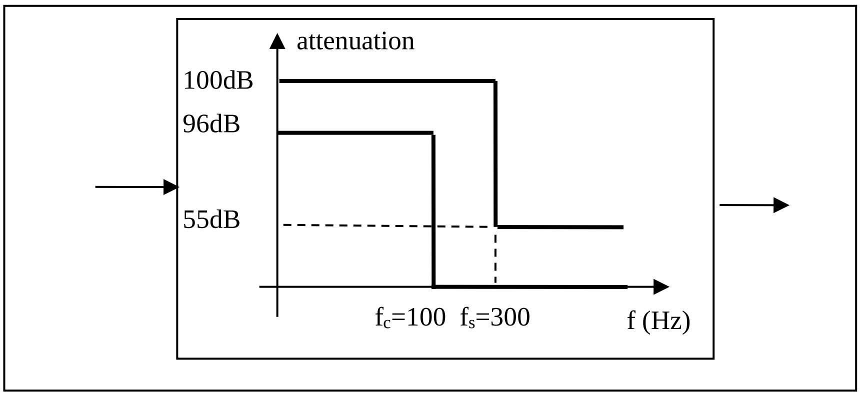


Figure 16: *Template of filter with static gain of 100dB*

Analyze **in AC mode** and determine:

- gain at low frequencies ($f \to 0$)
- cutoff frequency at -3dB
- the attenuation slope in dB/decade
- phase shift for $f \to \infty$
- Determine the cutoff frequency f^1_{-3dB} of the first-order filter placed at the top of the filter. Conclusion?

REMEMBER: It is important to remember that an active analog filter is synthesized from specifications which impose a template. This establishes the order (n) and the characteristic angular frequency/frequency of the filter (ω_0). In this TA, only the synthesis of a Butterworth-type filter has been covered as this is the only filter that can be applied using a simple analytical approach. In this case, and only in this case, the cutoff angular frequency is equal to the angular frequency at -3dB.

An active filter of order n consists of active blocks (1st and 2nd order) based on OP-AMPs. These blocks correspond to different types of "circuit" architecture (low-pass, high-pass, bandpass, etc.). Usually we use "Sallen-Key" cells, where determining the values of active components (R, C) must be based on knowing the characteristic angular frequency (ω_0) and the quality factor (Q) of each block.

Filter behavior can be analyzed from a point of view of frequency (harmonic regime) and time (transient or indicial state). We have also seen that group delay is an important notion as it determines the degree of distortion of a multi-frequency signal. In this context, the most interesting filter is the Bessel filter (known as linear phase) as it has a group delay which is virtually constant in its pass-band, unlike the Butterworth and Chebyshev filters.

Finally, particular attention should be paid to the use of OP-AMPs, which cannot be assumed to be ideal, especially when the specifications impose a large static gain (>80dB). They then impose their own frequency response (often 1st order) as their gain in differential mode (Ad) cannot always be considered as very high whatever the frequency.

III- Applied exercise *(taken from an S3 exam paper)*

Taking measurements from a filter, asymptotes have been drawn as shown in Figure 1.

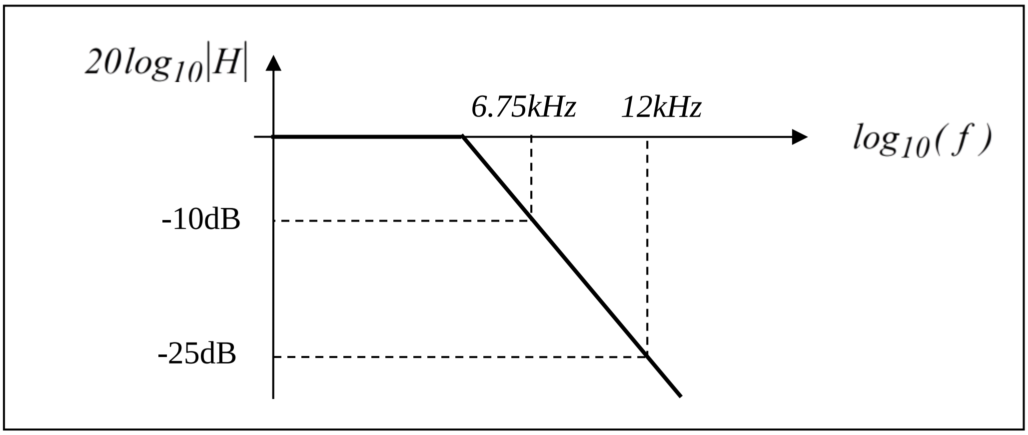


Figure 1: *Plot of the filter gain asymptotes*

Q1- What is the slope in $dB/decade$ when $f \rightarrow \infty$? Use this to establish the order of filter.

Q2- We consider the template of a low-pass Butterworth-type filter shown in Figure 2. Determine the order of this filter. The order will be the whole number immediately above the decimal value obtained. Remember the expression for the gain module of a Butterworth filter:

$$|H(j\omega)| = \frac{1}{\left(1+\left[\frac{\omega}{\omega_0}\right]^{2n}\right)^{1/2}}$$

where ω_0 represents the characteristic angular frequency.

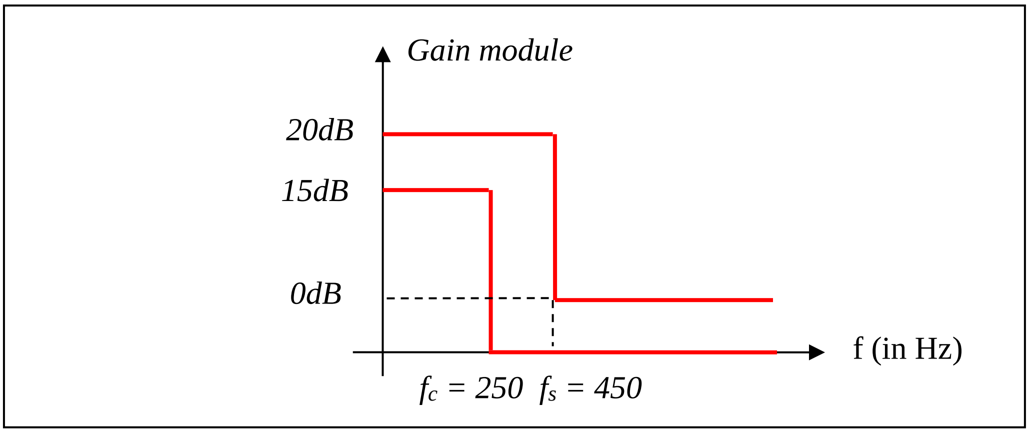


Figure 2: *Template of low-pass filter*

Q3- A low-pass fifth-order Bessel filter has a pass-band at *-3dB* equal to *1kHz* with a group delay of *2ms*. This filter is attacked by the following input signal: $e(t) = 2\cos(2\pi 30t) + 3\cos(2\pi 70t) + 0.5\cos(2\pi 10^4 t)$.

- What is the main characteristic of the group delay for a Bessel filter?
- Give the expression, with justification, of the output signal *s(t)* from the filter.

Tips for writing up your practical reports

The presentation of your practical experiment reports must respect a few crucial rules, in order to make them easier to read and assess. Each report will correspond to a practical assignment, and there is no need to copy out the text of the teaching materials. The notation system used in the teaching materials should be reproduced (e.g. if the output voltage is marked V_S in the teaching booklet, you should also mark it V_S in your report).

Your reports should adhere to the following structure:

<u>Cover page</u>:

- **Top left:** Names of both partners, with the partner responsible for drafting the report underlined (this responsibility should be alternated), along with the <u>date</u> of the practical work.
- **In the middle:** TP Title (e.g.: TP 2 - Frequency Spectrometry), framed.
- **Bottom of the page:** A short introduction (just a few lines) setting out the objectives of this practical assignment.

<u>Page(s) for presenting and analyzing your results</u>:

- The diagrams used to present measurements must be labelled and annotated, with inputs and outputs clearly marked.
- All figures must be accompanied by an explanatory caption, along with details of the measuring conditions as indicated in Figure 1. The number of each figure should also be incorporated into your text, ahead of the corresponding image.
  Example:
  "*… Figure 1 shows the variation in the output voltage Vout in reaction to input voltage V_{in} on TL081, measured at 25°C with an x10 probe….* "

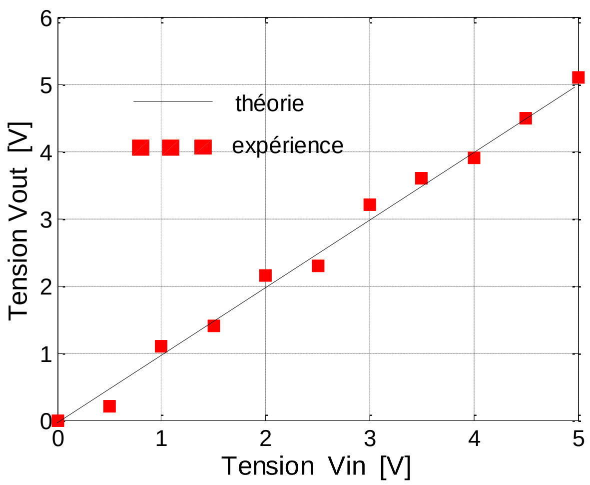


Figure 1: *Variation in the output voltage Vout in reaction to input voltage V_{in} on TL081 (25°C, x10 probe)*

- **Your report should be more than a simple succession of tests results with no explanation.** Every time you present a measurement or series of measurements, the results need to be backed up with analysis and a brief summary.

- Results of experiments should always specify the corresponding unit of measurement (example: $v = 2{,}2.10^{8}\ ms^{-1}$)

- MARK (/20): presentation of the report: /6, scientific and technical quality: /14 - You must hand in your report **<u>at the end of each session</u>** (unless otherwise agreed with your professor). Take care over your presentation! **You do not necessarily need to print out the signals or curves generated by your experiments. You can print them out if you are able to do so. If not, your reports should present your results in an intelligible fashion. <u>Please do not include photos taken with your mobile phone (the image resolution is not high enough)</u>.** - The grade average for practical work is based on all 6 TPs, and you will automatically receive a **<u>mark of 0/20</u>** if you **<u>fail to submit a report</u>** (unless your absence is justified). - Each practical session will last for 2.5 hours. You must have read the course material in advance and divided up the work with your partner in order to make optimal use of this time. The majority of these assignments include a **[BONUS] section** which can earn you extra points if you have time left over. If you answer the BONUS question correctly, you will gain 4 extra marks. - It is essential that you bring your calculator, lecture notes and tutored assignment notes to help you with your analyses and reports. Failure to abide by these rules will lead to **<u>automatic exclusion</u>** from the practical assignment.

Practical assignement:

Analog filter

I- Analysis of the Bessel filter and comparison with the Butterworth filter

II- Influence of the op-amp characteristics on the performance of the Butterworth filter

III- Experimental and simulated study of an ADSL filter [BONUS]

As seen in the tutored assignment: To synthesize an active analog filter we need to refer to the technical specifications, which specify a filter shape. This determines the order (n) and pulse frequency which are characteristic of the filter (ω_0). In this tutorial we will limit ourselves to the synthesis of Butterworth-type filters, the only type of filter which allows for a simple analytical approach. With this type of filter, the cut-off frequency is equivalent to a pulse of -3dB.

An active filter of order n is made up of active blocks (1st and 2nd order) based on operations amplifiers (op-amps). These blocks correspond to different types of circuit architecture (low-pass, high-pass, pass-band etc.). We generally use Sallen-Key cells, for which the passive component values (R, C) must be determined with reference to the characteristic pulse frequency (ω_0) and quality factor (Q) for each block.

The behavior of the filters can be analyzed from a frequential (harmonic mode) and temporal (transient or indexed mode) perspective. As we have seen, group delay is an important factor to bear in mind because it will determine the degree of distortion affecting a multi-frequency signal. In this context, the most useful filter is the Bessel (or linear phase) filter, because its group delay is virtually constant across the pass-band, which is not true of the Butterworth or Tchebychev filters.

Last but not least, special attention must be paid to the use of op-amps, which cannot be supposed to be perfect - especially when the technical specifications require heavy static gain (>80dB). They thus impose their own frequency response (often first order), as their gain in automatic differentiation mode (AD) cannot always be considered as very high regardless of the frequency.

The TINA-TI simulation software

Tina-TI (available to download from Texas Instruments www.ti.com is a program used to simulate electronic circuits. It includes a Spice model library of op-amps, instrument amplifiers, comparators, voltage regulators etc. developed by Texas Instruments. Once the circuit diagram has been entered, various analyses are available.

- DC analysis
- AC analysis
- Transient analysis
- Fourier analysis
- Noise analysis

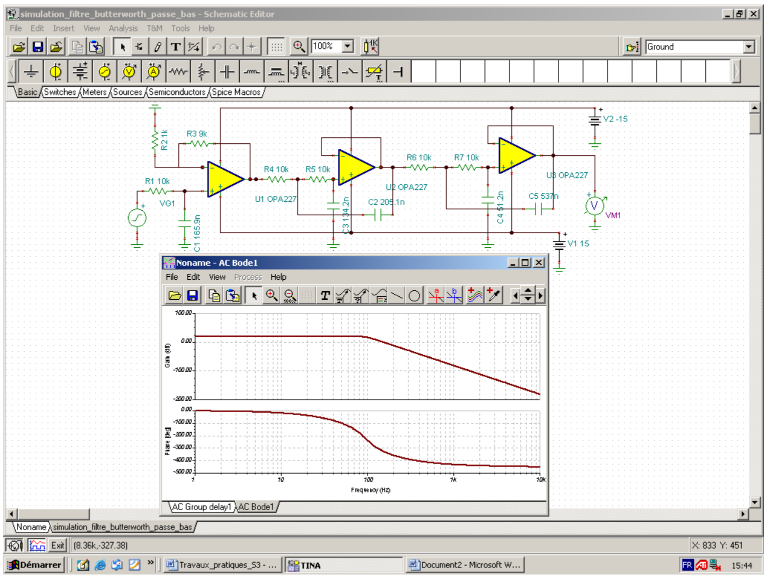


Figure 1*: Example of a window in TINA-TI: entering the circuit diagram and results of the AC Analysis simulation (Bode)*

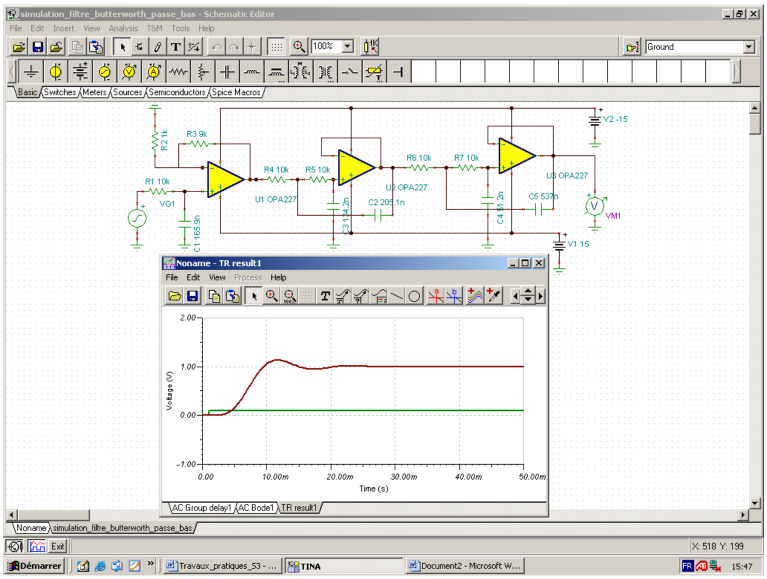


Figure 2*: Example of a window in TINA-TI: entering the circuit diagram and results of the Transient Analysis*

The program is very easy to use: to select a component, click OK in the 'Select instrument' window and drag it into the workspace. To adjust the settings for a component, double click on it and fill in the appropriate fields. Join up the components and proceed with the analyses mentioned above.

N.B.: TINA-TI cannot simulate circuits unless they contain at least one integrated circuit, but this is not a problem as we can just add a simple component such as a tracker in order to analyze a circuit containing only passive components.

I- Analysis of the Bessel filter and comparison with the Butterworth filter

Modify the capacitance values to simulate a 5th order Bessel filter with a **cut-off frequency of -3dB identical** to that of the Butterworth filter.

The aim of this exercise is to compare the results obtained using this filter with those from the tutorial where we used a Butterworth filter. **If you didn't have time to finish these simulations during the tutorial, you can do it during the practical session.**

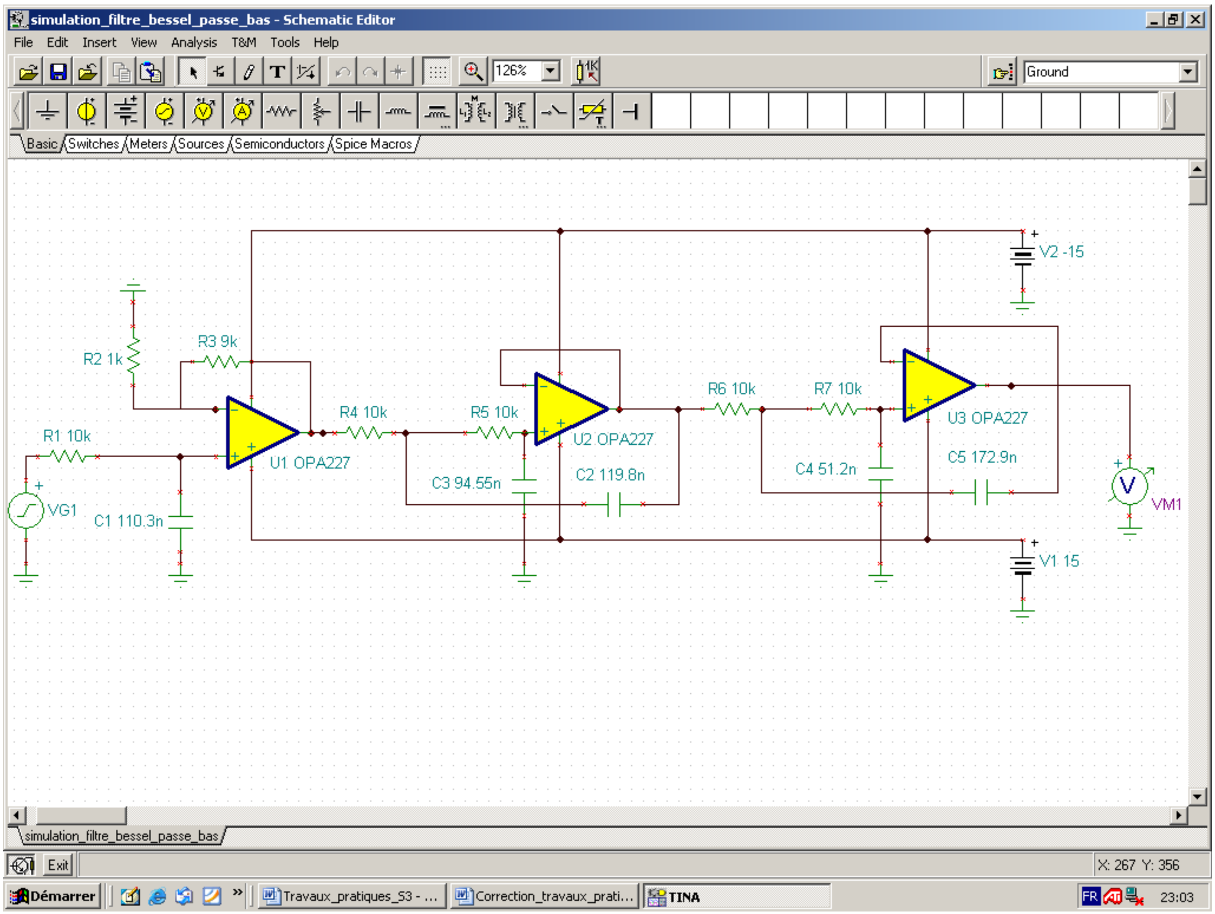


Figure 1: *Electrical diagram for the 5th order Bessel filter*

Harmonic analysis

- Menu bar → Analysis → AC Analysis → AC Transfer Characteristic

- Fill in the fields (Start Frequency, End Frequency, Number of points, Sweep type Logarithmic, Diagram Amplitude & Phase)

Question 1: Using the zoom function and cursors, determine:

- the gain at low frequencies ($f \to 0$)
- the cutoff frequency f_{-3dB}
- the attenuation slop in dB/decades for $f \to \infty$
- the phase shift for $f \to \infty$
- the cutoff frequency f^1_{-3dB} for the first order filter positioned at the head of the filter. Give the value of k in $f^1_{-3dB} = kf_{-3dB}$. Does this correspond to what you expected to see? How do you interpret these results?

Question 2: Using the copy and paste function, plot the **gain curves** from the Butterworth (as seen in the TD) and Bessel filters on the same graph, in the range 0 – 400Hz. Which filter comes closest to "ideal" gain?

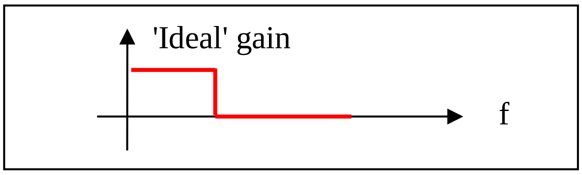


Figure 2: *'Ideal' gain*

Question 3: Using the copy and paste function, plot the **phase** of the Butterworth and Bessel filters on the same graph, in the range 0 – 100Hz. Use a linear scale for the frequencies. Which filter comes closest to "ideal" gain?

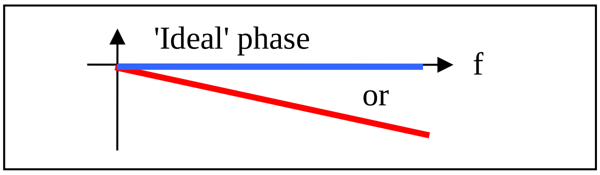


Figure 3: *'Ideal' phase*

Question 4: **Group delay**

Using the copy and paste function, plot the **group delay** of the Butterworth and Bessel filters on the same graph, in the range 0 – 100Hz. Use a linear scale for the frequencies.

Does the group delay vary with the pass-band? What is its value? What conclusion can you draw from this?

To answer this question, refer to your lecture notes and the tutored assignment.

Question 5: **Step response from the filter**

Take the settling time at 1%. What is the delay at 50%?

N.B.: *You will need to check that the delay is of the same order of magnitude as the group delay measured above.*

Question 6: Using the copy and paste function, plot the **step response** of the Butterworth and Bessel filters on the same graph, for the period 0 – 20ms. What do you notice?

II- Influence of the op-amp characteristics on the performance of the Butterworth filter

Modify the diagram to obtain a Butterworth filter which fits the shape shown in Figure 4.

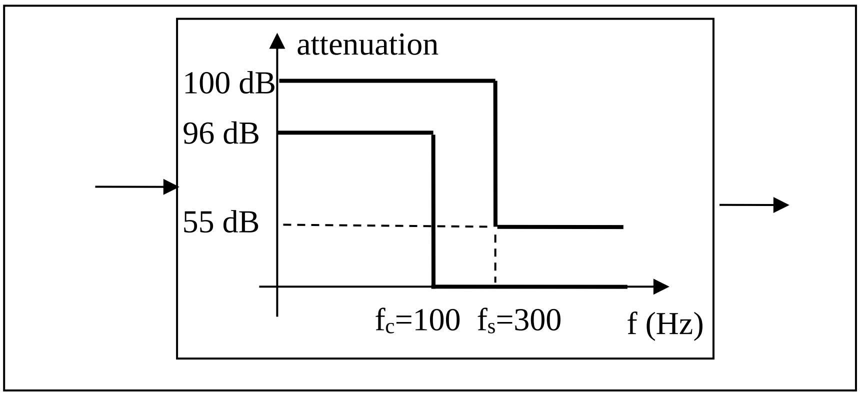


Figure 4: *Filter shape, gain of 100 dB for* $f \to 0$

Question 7: Analyze the harmonics and calculate:

- the gain at low frequencies ($f \to 0$)
- the cut-off frequency at -3 dB
- the attenuation slop in dB/decades
- the phase shift for $f \to \infty$

Determine the cut-off frequency f^1_{-3dB} of the first order filter placed ahead of the filter and explain why the results of the simulation are different from what you had expected.

III- Experimental and simulated study of an ADSL filter [BONUS]

These filters are made up of passive elements (coils and capacitors), as seen in the commercially-available filter shown in Figure 5.

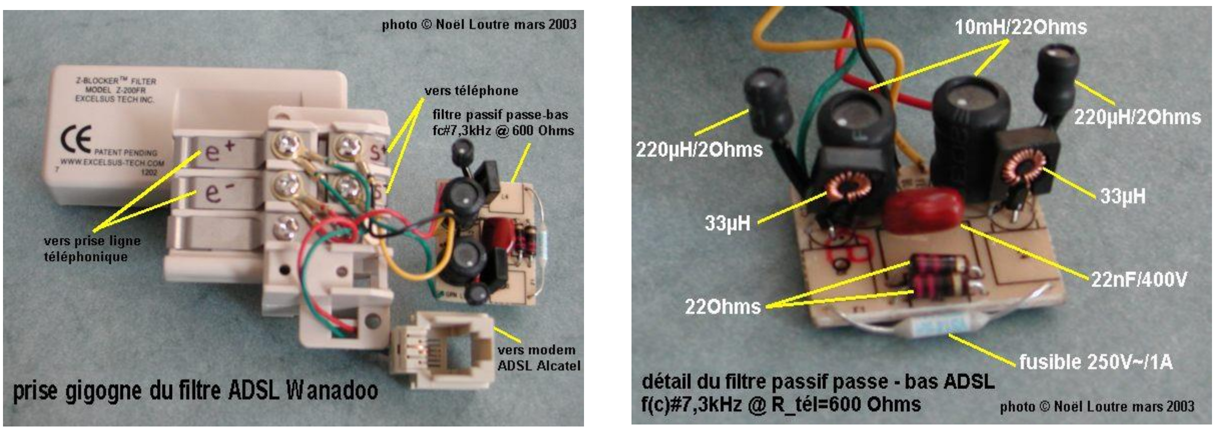


Figure 5: *ADSL filter (via www.loutre.org)*

Having read the appendix, what is the role of the ADSL low-pass filter and what is its approximate cut-off frequency?

Experiment

Test a standard commercial ADSL filter with an impedance generator of 50 Ω, and the mathematical functions of your oscilloscope. The circuit diagram for the filter is given in Figure 6.

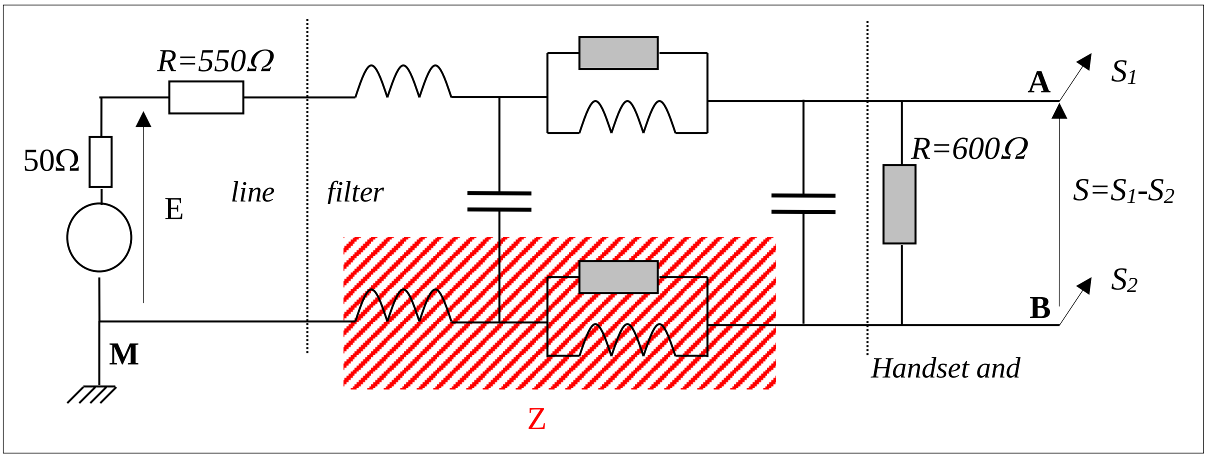


Figure 6: *Circuit diagram of the ADSL filter*

To determine the ratio $\frac{S}{E}$, we need to take a **measurement <u>in differential mode</u>**. If we connect the oscilloscope to the terminals with resistance $R = 600\Omega$, the impedance Z is short-circuited.

Procedure to follow to obtain $\frac{S}{E} = \frac{S_1 - S_2}{E}$:

- Signal $S_1 = V_A - V_M$ is connected to input Y_1 of the oscilloscope.
- Signal $S_2 = V_B - V_M$ is connected to input Y_2 of the oscilloscope.
- Use the 'maths' function of the oscilloscope to calculate the difference $S_1 - S_2$.

<u>Question 8</u>: Plot the graph $20\log_{10}\left(\frac{S}{E}\right)$ against $\log_{10}(f)$ for the range $0 - 50\text{kHz}$, then determine:

- the static gain for $f \to 0$
- the cut-off frequency at -3 dB
- the attenuation slop in dB/decades for $f \to \infty$

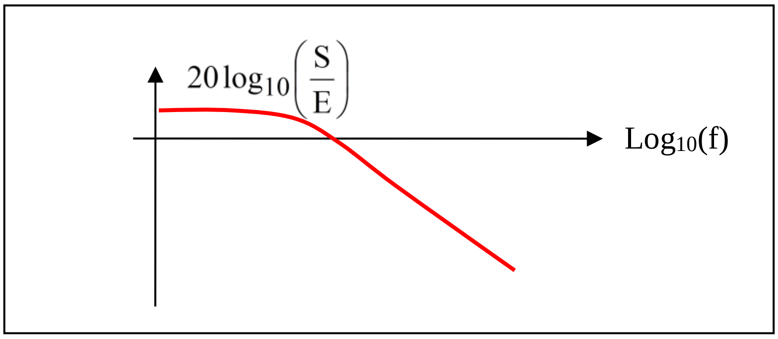


Figure 7: $20\log_{10}\left(\frac{S}{E}\right)$ *against* $\log_{10}(f)$

Validation through simulation

Enter the simplified ADSL filter circuit given in Figure 8.

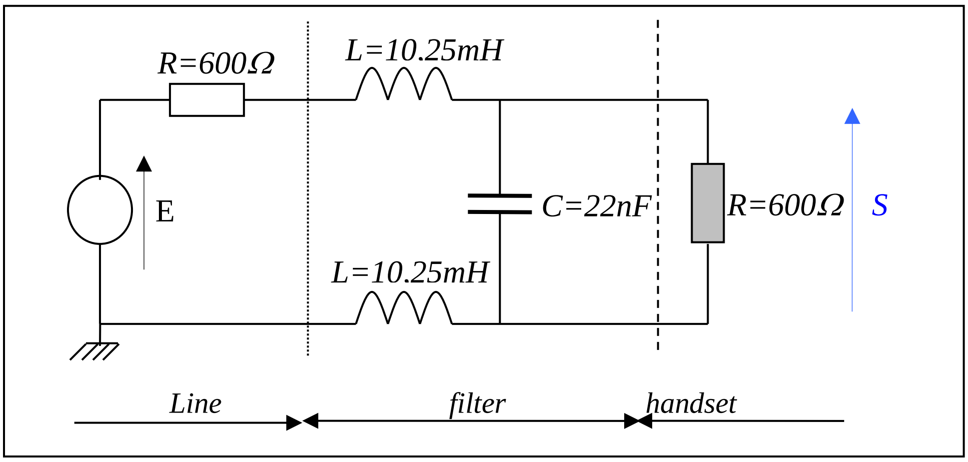


Figure 8: *Simplified circuit diagram of the ADSL filter*

<u>Question 9</u>: To analyze this circuit in harmonic mode, you will need to add an active component: connect an INA105 difference amplifier to the 600Ω terminals (Spice macro → Difference Amplifiers → INA105E).

- Plot the gain and phase for 0-100kHz
- Determine the gain for $f \to 0$
- Cut-off frequency at -3 dB
- the attenuation slop in dB/decades for $f \to \infty$

Appendix - the ADSL filter

When ADSL and PSTN (public switched telephone network) work at the same line at the same time, the electronics inside a normal telephone can be problem for high frequency ADSL signals: the ADSL signals can be attenuated (high capacitance on telephone input, possible resonances inside telephone, impedance mismatch) and ADSL signals can be heard as noise on some telephones (phone electronics demodulates high frequency signal outside it's operating range to voice frequency noise). In order to keep these systems apart and stop them interfering with each other it is necessary to separate the two components from the telephone line in your home.

The signal to telephone output is generally just low-pass filtered so that voice frequencies (frequencies up to 3.4 kHz) get nicely though, but higher frequencies gets filtered. This filtering generally consists of LC low-pass filter designed to some suitable operating frequency between 4 and 20 kHz (between voice and ADSL bands). This kind of filter causes that the high frequencies of the ADSL signal will be severely attenuated (usually by at least 30dB with a good filter) so the signal reaching your telephone equipment does not contain such amount of high frequency signals that could cause noise. The telephone LC filter is also designed in such way that the filter impedance towards the line that carries ADSL signals is high at the high frequencies, meaning that those telephone equipment (and cables related to them) look like they are "disconnected from the main line" at ADSL high frequencies.

General design specifications for an ADLS filter should be something like this:

** Return loss at voice frequencies (against 600 ohms) should be good enough.*

** Should not alter voice band frequency response too much*

** Insertion loss at voice frequencies should be good enough, which means that the filter should not have too high series resistance (commercial filters seems to have between 50 and 100 ohms for whole loop resistance)*

** Filter must pass the POTS tip-to-ring dc voltages (typically 0-72V)*

** Filter must pass ring voltages well (40V to 80V rms at any frequency from15.3Hz to 68Hz with a DC component in the range from 0V to 72V)*

** All requirements must be met in the presence of POTS loop currents (usually around 0-40 mA, can be up to 120 mA in some cases)*